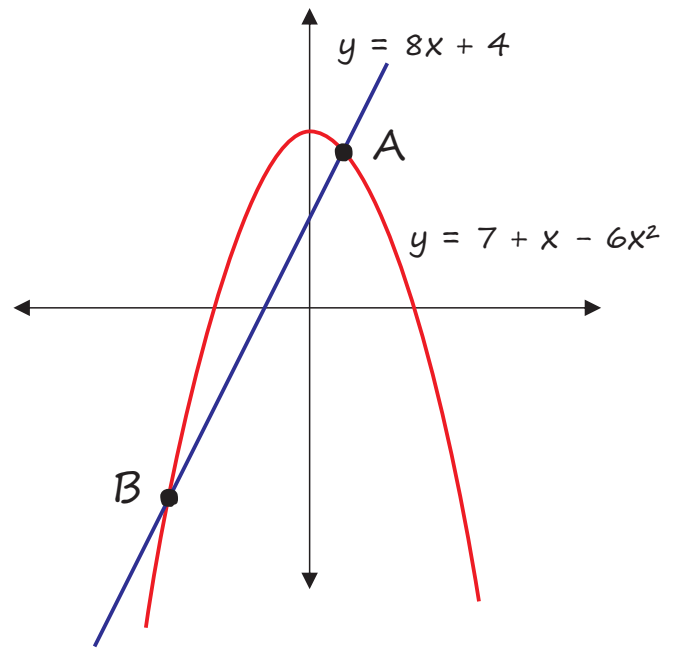
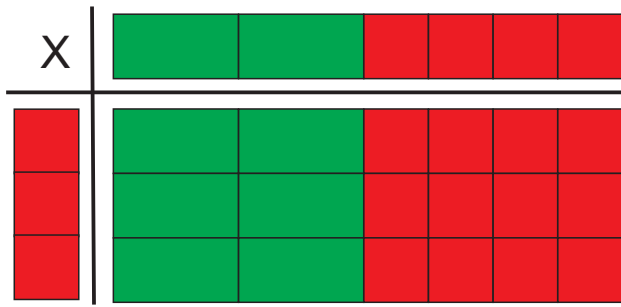


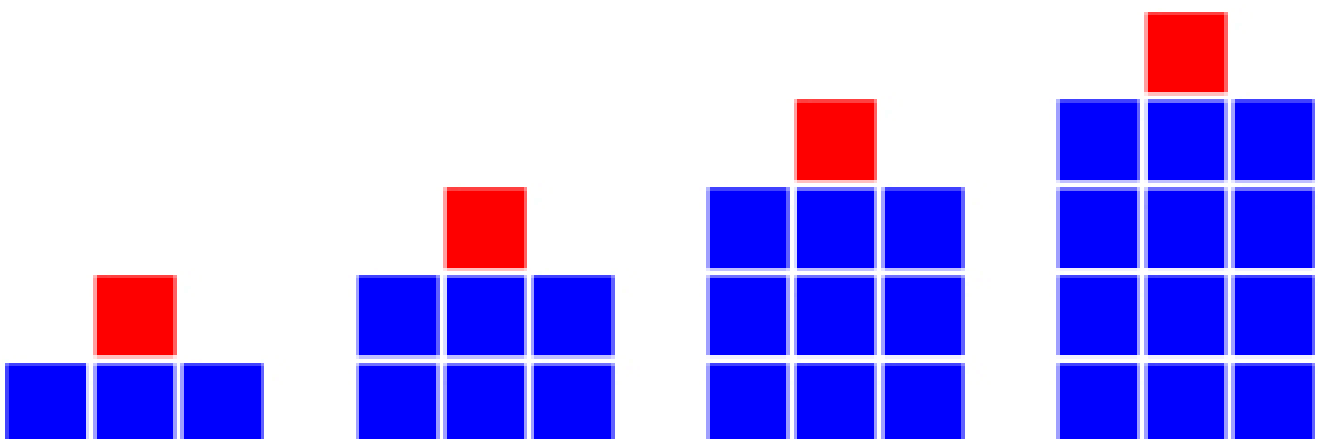
Expand $3(2x + 4) = 6x + 12$



THE ALGEBRA PROJECT

Kim Freeman

THE ANSWERS



The Algebra Project
Kim Freeman

Published in 2021 by:
Mahobe Resources (NZ) Ltd
P.O. Box 109-760
Newmarket, Auckland
New Zealand



www.mathscentre.co.nz

© Mahobe Resources (NZ) Ltd
ISBN(13) 978-1-877489-31-0

This Book has been provided by Mahobe Resources (NZ) Ltd to *The New Zealand Centre of Mathematics*. School teachers, University lecturers, and their students are able to freely download this book from *The New Zealand Centre of Mathematics* website www.mathscentre.co.nz. Electronic copies of the complete Book may not be copied or sold. Students have permission to print one copy for their personal use. Any photocopying by teachers must be for training or educational purposes and must be recorded and carried out in accordance with Copyright Licensing Ltd guidelines. The content presented within the book represents the views of the publisher and the author as at the date of publication. Because of the rate with which conditions change, the publisher and his contributors reserve the right to alter and update the contents of the book at any time based on the new conditions. This Book is for informational purposes only and the publisher and his contributors do not accept any responsibilities for any liabilities resulting from the use of the information within. While every attempt has been made to verify the content provided, neither the publisher nor his contributors and partners assume any responsibility for errors, inaccuracies or omissions. All rights reserved. All the views expressed in this book are those of the author at the time of writing.

Algebra Like Terms

Simplify all these expressions

- $a + a + a = 3a$
- $x + x = 2x$
- $p + p + p + p + p = 5p$
- $y + y + y = 3y$
- $i + i + i + i + i + i + i = 7i$
- $k + k + k + k = 4k$
- $4x + x = 5x$
- $10x + 2x = 12x$
- $7y + 3y + y = 11y$
- $8j + j + j = 10j$
- $15d + 3d + 2d = 20d$
- $12a + 5a + 2a = 19a$
- $13c - 4c = 9c$
- $17u - 8u = 9u$
- $8g - 10g = -2g$
- $6r + 6r - r = 11r$
- $5t - 3t + 2t = 4t$
- $10h + 8h - 12h = 6h$
- $6x - 8x + x = -x$
- $-8y + 6y - 5y = -7y$
- $3r - 8r + 2r = -3r$
- $-5w - 6w = -11w$
- $3y - 6y + y = -2y$
- $f - 6f - 2f = -7f$
- $4\pi - 7\pi + \pi = -2\pi$
- $3e + 8s - 2s - e = 2e + 6s$
- $9i + 4j - 6i - 2j = 3i - 2j$
- $-3t - 5s + 5t - 2s = 2t - 7s$
- $4d + 2e - 4e - 8d = -4d - 2e$
- $-4f + 3h - 7h + 5f = f - 4h$
- $11 + 3n + 6 + 9n = 17 + 12n$
- $8 + 3d - 6 - 5d = 2 - 2d$
- $r - 5 - 5r + 10 = -4r + 5$
- $g - 12g + 7h = -11g + 7h$
- $7j + 5k - 2j - 5k = 5j$
- $4w - 6x + 8w - 6w = 6w - 12x$
- $-4m + 6n - 10n + 4n = -4m$
- $5j + 5k = 5j + 5k$
- $2s - 7t - 10s - 2t = -8s - 9t$
- $3x^2 + 2x^2 - 3y = 5x^2 - 3y$
- $5x + 6y - 7y - 8x = -3x - y$
- $-2q + 2p - 7p + 8q = 6q - 5p$
- $14ab - 7b - 3ab + 8b = 11ab + b$
- $-2xy + 5x - 8xy - 9x = -10xy - 4x$
- $8y^2 - 2y + 5y - 6y^2 = 2y^2 + 3y$
- $16gh + 8g - 7gh = 9gh + 8g$
- $6bg - 2b - 5bg = bg - 2b$
- $-6mn - 12n + 8n = -6mn - 4n$
- $-5xy + 12yx = 7xy$
- $-9ab + 8ab + 9b = -ab + 9b$

Algebra Expressions

Simplify all these expressions

- $a \times a = a^2$
- $y \times y \times y \times y = y^4$
- $p \times p \times p = p^3$
- $w \times w \times w \times w \times w = w^5$
- $i \times i \times i \times i \times i \times i \times i = i^7$
- $k \times k \times k \times k = k^4$
- $5a \times b = 5ab$
- $8f \times 2x = 16fx$
- $9y \times 3z = 27yz$
- $5j \times 3k = 15jk$
- $7d \times 3e = 21de$
- $12a \times 5b = 60ab$
- $15c \times 2c = 30c^2$
- $7u \times 8u = 56u^2$
- $8g \times 5g = 40g^2$
- $6r^2 \times r = 6r^3$
- $5p^3q \times 3p = 15p^4q$
- $10h^2j \times 8j = 80h^2j^2$
- $6b^2 \times 9b^3 \times b = 54b^6$
- $-8y \times 6y^2 = -48y^3$
- $4r^2 \times 9r^2 \times r = 36r^5$
- $-5w^3 \times -6w^2 = 30w^5$
- $3y^2 \times 6y^2 \times y = 18y^5$
- $2g^3f \times 6gf^2 = 12g^4f^3$
- $4p^2qr \times 5r \times 2p = 40p^3r^2q$

- $(y^2)^3 = y^6$
- $(5a)^2 = 25a^2$
- $(2y)^5 = 32y^5$
- $(3y)^2 = 9y^2$
- $(5xy)^3 = 125x^3y^3$
- $-(2x)^2 = -4x^2$
- $(-2x)^2 = 4x^2$
- $(x^2)^7 = x^{14}$
- $(k^5h^3)^2 = k^{10}h^6$
- $w^0 = 1$
- $\sqrt{49a^{12}} = 7a^6$
- $\sqrt{x^{100}} = x^{50}$
- $\sqrt{9a^{10}b^6} = 3a^5b^3$
- $\sqrt{100g^2h^{100}} = 10gh^{50}$

If $a = 2$, $b = 5$, $c = 10$ calculate:

- $(a + b)^2 = 49$
- $4b^2 - (3a)^2 = 64$
- $(a^2)^2 = 16$
- $c^2 - a^2 + b^2 = 121$
- $(2a^2)^3 = 256$
- $2c^2 - 2b^2 = 150$
- $4(a^4 - 10) = 24$
- $b(5c - a^2) = 230$
- $3a + 4c^3 = 4006$
- $\sqrt{c^2} = 10$

Algebraic Fractions

Simplify all these fractions!

$$\blacksquare \frac{3}{x} + \frac{5}{x} = \frac{8}{x}$$

$$\blacksquare \frac{7}{w} - \frac{5}{w} = \frac{2}{w}$$

$$\blacksquare \frac{x}{5} + \frac{x}{15} = \frac{4x}{15}$$

$$\blacksquare \frac{b}{3} - \frac{b}{6} = \frac{b}{6}$$

$$\blacksquare \frac{3}{x^2} - \frac{2}{x} = \frac{3 - 2x}{x^2}$$

$$\blacksquare \frac{2}{x^2} + \frac{5}{x^3} = \frac{2x + 5}{x^3}$$

$$\blacksquare \frac{7}{x^3} + \frac{4}{x} = \frac{7 + 4x^2}{x^3}$$

$$\blacksquare \frac{s}{4} + \frac{5}{3} = \frac{3s + 20}{12}$$

$$\blacksquare \frac{x}{3} - \frac{x}{7} = \frac{4x}{21}$$

$$\blacksquare \frac{r}{5} + \frac{r}{8} = \frac{13r}{40}$$

$$\blacksquare \frac{x}{a} + \frac{2}{d} = \frac{xd + 2a}{ad}$$

$$\blacksquare \frac{4}{m} - \frac{x}{n} = \frac{4n - xm}{mn}$$

$$\blacksquare \frac{5}{a} + \frac{c}{c} = \frac{5c + ac}{ac} \text{ or } \frac{5+a}{a} \text{ or } 1\frac{5}{a}$$

$$\blacksquare \frac{7}{xy} + \frac{2}{y} = \frac{7y + 2xy}{xy^2} \text{ or } \frac{7 + 2x}{xy}$$

$$\blacksquare \frac{5}{ab} - \frac{3}{a} = \frac{5a - 3ab}{a^2b} \text{ or } \frac{5 - 3b}{ab}$$

$$\blacksquare \frac{7}{x} + \frac{7}{xy} = \frac{7xy + 7x}{x^2y} \text{ or } \frac{7y - 7}{xy}$$

$$\blacksquare \frac{3h}{gh} = \frac{3}{g}$$

$$\blacksquare \frac{12m}{4n} = \frac{3m}{n}$$

$$\blacksquare \frac{6x}{18y} = \frac{x}{3y}$$

$$\blacksquare \frac{mp}{6m} = \frac{p}{6}$$

$$\blacksquare \frac{3m^4}{6m^2} = \frac{m^2}{2}$$

$$\blacksquare \frac{2w^5}{8w^2} = \frac{w^3}{4}$$

$$\blacksquare \frac{8x^5}{4x^3} = 2x^2$$

$$\blacksquare \frac{50r^4}{5r^2} = 10r^2$$

$$\blacksquare \frac{7e^7}{21e^3} = \frac{e^4}{3}$$

$$\blacksquare \frac{5m^5}{25m^2} = \frac{m^3}{5}$$

$$\blacksquare \frac{12r^9}{3r^3} = 4r^6$$

$$\blacksquare \frac{27m^4}{9m} = 3m^3$$

$$\blacksquare \frac{8a}{15n} \times \frac{5c}{4a} = \frac{2c}{3n}$$

$$\blacksquare \frac{3c}{4e} \times \frac{2e^3}{4c} = \frac{3e^2}{8}$$

$$\blacksquare \frac{5ab}{2a} \times \frac{5a}{10ab} = \frac{5}{4}$$

$$\blacksquare \frac{3}{4b} \times \frac{15}{8b^2} = \frac{45}{32b^3}$$

$$\blacksquare \frac{3ab^2}{5cd} \times \frac{5c}{7a^2} = \frac{3b^2}{7ad}$$

If $a = \frac{1}{2}$, $b = 4$, $c = 10$ calculate:

$$\blacksquare \frac{2a + b}{c} = \frac{1}{2}$$

$$\blacksquare \frac{4a^2}{b} = \frac{1}{4}$$

$$\blacksquare \frac{ac}{bc} \times \frac{bc}{ac} = 1$$

$$\blacksquare \frac{c}{b} \times \frac{a}{b} = \frac{15}{16}$$

$$\blacksquare \frac{c}{b} \div \frac{a}{b} = 20$$

$$\blacksquare \frac{4a + c}{b} = 3$$

$$\blacksquare a(c^2 + b^2) = 58$$

Algebra Expansions

Expand all these expressions!

- ▣ $2(6 + 3x) = 12 + 6x$
- ▣ $5(3a - 5) = 15a - 25$
- ▣ $10(2b + 4) = 20b + 40$
- ▣ $4(5c - 7) = 20c - 28$
- ▣ $7(4d + 5e) = 28d + 35e$
- ▣ $4f(f + 2) = 4f^2 + 8f$
- ▣ $5g(2g - 6) = 10g^2 - 30g$
- ▣ $6h(h - 5) = 6h^2 - 30h$
- ▣ $5x(x^2 - x + 2)$
 $= 5x^3 - 5x^2 + 10x$
- ▣ $3(a^2 - 18a + 4)$
 $= 3a^2 - 54a + 12$
- ▣ $2(a + 2b) + (a - b)$
 $= 2a + 4b + a - b$
 $= 3a + 3b$
- ▣ $4(c - 3d) - 3(c + 2d)$
 $= 4c - 12d - 3c - 6d$
 $= c - 18d$
- ▣ $3(2e - f) + 2(e - 2f)$
 $= 6e - 3f + 2e - 4f$
 $= 8e - 7f$
- ▣ $3x(x + 8) - x(2x - 5)$
 $= 3x^2 + 24x - 2x^2 - 5x$
 $= x^2 + 19x$

- ▣ $(x + 5)(x + 1)$
 $= x^2 + 6x + 5$
- ▣ $(x - 7)(x - 2)$
 $= x^2 - 9x + 14$
- ▣ $(a + 6)(a - 4)$
 $= a^2 + 2a - 24$
- ▣ $(x - 3)(x + 8)$
 $= x^2 + 5x - 24$
- ▣ $(x - 9)(x + 9)$
 $= x^2 - 81$
- ▣ $(x + 6)^2$
 $= x^2 + 12x + 36$
- ▣ $(x + 9)(x + 4)$
 $= x^2 + 13x + 36$
- ▣ $(x - 12)(x + 1)$
 $= x^2 - 11x - 12$
- ▣ $(x + 11)(x - 5)$
 $= x^2 + 6x - 55$
- ▣ $(x - 15)(x - 3)$
 $= x^2 - 18x + 45$
- ▣ $(7 - x)(7 + x)$
 $= 49 - x^2$
- ▣ $(9 - x)^2$
 $= 81 - 18x + x^2$
- ▣ $(3x + 2)(2x - 1)$
 $= 6x^2 + x - 2$
- ▣ $(4x - 7)(x + 5)$
 $= 4x^2 + 13x - 35$

Quadratic Expansions

Want extra practise? Expand all these expressions!

- $$\begin{aligned} \square & (x + 3)(x - 8) \\ & = x^2 - 5x - 24 \end{aligned}$$
- $$\begin{aligned} \square & (x + 12)(x + 10) \\ & = x^2 + 22x + 120 \end{aligned}$$
- $$\begin{aligned} \square & (x - 9)(x - 8) \\ & = x^2 - 17x + 72 \end{aligned}$$
- $$\begin{aligned} \square & (x - 7)(x + 11) \\ & = x^2 + 4x - 77 \end{aligned}$$
- $$\begin{aligned} \square & (x + 8)(x + 1) \\ & = x^2 + 9x + 8 \end{aligned}$$
- $$\begin{aligned} \square & (x - 2)(x + 6) \\ & = x^2 + 4x - 12 \end{aligned}$$
- $$\begin{aligned} \square & (x - 5)(x - 13) \\ & = x^2 - 18x + 65 \end{aligned}$$
- $$\begin{aligned} \square & (x + 15)(x - 4) \\ & = x^2 + 11x - 60 \end{aligned}$$
- $$\begin{aligned} \square & (x + 3)(6 - x) \\ & = -x^2 + 3x + 18 \end{aligned}$$
- $$\begin{aligned} \square & (2 - x)(3 - x) \\ & = 6 - 5x + x^2 \end{aligned}$$
- $$\begin{aligned} \square & (10 - x)(4 + x) \\ & = 40 + 6x - x^2 \end{aligned}$$
- $$\begin{aligned} \square & (x + 8)(5 + x) \\ & = x^2 + 13x + 40 \end{aligned}$$
- $$\begin{aligned} \square & (2x + 3)(3x + 4) \\ & = 6x^2 + 17x + 12 \end{aligned}$$
- $$\begin{aligned} \square & (5x + 1)(x - 2) \\ & = 5x^2 - 9x - 2 \end{aligned}$$
- $$\begin{aligned} \square & (4x - 5)(2x - 3) \\ & = 8x^2 - 22x + 15 \end{aligned}$$
- $$\begin{aligned} \square & (1 - 2x)(3x + 7) \\ & = -6x^2 - 11x + 7 \end{aligned}$$
- $$\begin{aligned} \square & (x - 5)(x + 5) \\ & = x^2 - 25 \end{aligned}$$
- $$\begin{aligned} \square & (1 + 2x)(1 - 2x) \\ & = 1 - 4x^2 \end{aligned}$$
- $$\begin{aligned} \square & (3 - 2x)(2x + 3) \\ & = 9 - 4x^2 \end{aligned}$$
- $$\begin{aligned} \square & (x + 7)^2 \\ & = x^2 + 14x + 49 \end{aligned}$$
- $$\begin{aligned} \square & (x - 5)^2 \\ & = x^2 - 10x + 25 \end{aligned}$$
- $$\begin{aligned} \square & (3x - 1)^2 \\ & = 9x^2 - 6x + 1 \end{aligned}$$
- $$\begin{aligned} \square & (2 + 4x)^2 \\ & = 4 + 16x + 16x^2 \end{aligned}$$
- $$\begin{aligned} \square & (2x + 3y)^2 \\ & = 4x^2 + 12xy + 9y^2 \end{aligned}$$
- $$\begin{aligned} \square & (6x - 2y)^2 \\ & = 36x^2 - 24xy + 4y^2 \end{aligned}$$
- $$\begin{aligned} \square & 2(3x - 5)^2 \\ & = 18x^2 - 60x + 50 \end{aligned}$$

Factorising

Factorise all these expressions!

$$\blacktriangleright 10a - 15b = 5(2a - 3b)$$

$$\blacktriangleright 3xy - 12xz = 3x(y - 4z)$$

$$\blacktriangleright 4\pi x - 4\pi y = 4\pi(x - y)$$

$$\blacktriangleright 4a^2 + 3a = a(4a + 3)$$

$$\blacktriangleright x^3 + x^2y = x^2(x + y)$$

$$\blacktriangleright a^2 - 6y = a^2 - 6y$$

$$\blacktriangleright 2x^3 + 2xy^2 = 2x(x^2 + y^2)$$

$$\blacktriangleright y^2 + y^3 = y^2(1 + y)$$

$$\blacktriangleright x^2 + 2xy - x = x(x + 2y - 1)$$

$$\blacktriangleright 36 + 6x^3 = 6(6 + x^3)$$

$$\blacktriangleright 2\pi r^2 + \pi rh = \pi r(2r + h)$$

$$\blacktriangleright x^2y - xy^2 = xy(x - y)$$

$$\blacktriangleright 6a^2 - 12a = 6a(a - 2)$$

$$\blacktriangleright 5x^2 - 10xy = 5x(x - 2y)$$

$$\blacktriangleright 4a^2b + 8ab^2 = 4ab(a + 2b)$$

$$\blacktriangleright 21x^3 - 14x^2 = 7x^2(3x - 2)$$

$$\blacktriangleright t^2 + 4t = t(t + 4)$$

$$\blacktriangleright 20x + 10x - 5y + 25y$$

$$= 10(3x + 2y)$$

$$\blacktriangleright 14a^2b + 21ab^2$$

$$= 7ab(2a + 3b)$$

$$\blacktriangleright 3c + 6d + 12e$$

$$= 3(c + 2d + 4e)$$

$$\blacktriangleright 14a - 21b = 7(2a - 3b)$$

$$\blacktriangleright 3xy + 9yz = 3y(x + 3z)$$

$$\blacktriangleright 2\pi d - 2\pi r = 2\pi(d - r)$$

$$\blacktriangleright 15x^2 - 20x = 5x(3x^2 - 4)$$

$$\blacktriangleright r^3 + r^2 = r^2(r + 1)$$

$$\blacktriangleright 9 + 3n^2 = 3(3 + n^2)$$

$$\blacktriangleright 2x^2 + 2xy = 2x(x + y)$$

$$\blacktriangleright 5x^2 + 5y^2 = 5(x^2 + y^2)$$

$$\blacktriangleright 3x^3 + xy = x(3x^2 + y)$$

$$\blacktriangleright 4ab - 8bc = 4b(a - 2c)$$

$$\begin{aligned} \blacktriangleright ax + bx + ay + by \\ &= x(a + b) + y(a + b) \\ &= (x + y)(a + b) \end{aligned}$$

$$\begin{aligned} \blacktriangleright 1 + x + y + xy \\ &= 1(1 + x) + y(1 + x) \\ &= (1 + y)(1 + x) \end{aligned}$$

$$\begin{aligned} \blacktriangleright 2xy + 6y - x - 3 \\ &= 2y(x + 3) - 1(x + 3) \\ &= (2y - 1)(x + 3) \end{aligned}$$

$$\begin{aligned} \blacktriangleright a^2 + ab - ac - bc \\ &= a(a + b) - c(a + b) \\ &= (a - c)(a + b) \end{aligned}$$

$$\begin{aligned} \blacktriangleright 2ax - 4ay - 3bx + 6by \\ &= 2a(x - 2y) - 3b(x - 2y) \\ &= (2a - 3b)(x - 2y) \end{aligned}$$

Quadratic Factoring 1

Factorise all these quadratic expressions!

- $x^2 + 11x + 24$
= $(x + 8)(x + 3)$
 - $x^2 + 11x + 18$
= $(x + 9)(x + 2)$
 - $x^2 - 100$
= $(x + 10)(x - 10)$
 - $x^2 + 9x - 36$
= $(x + 12)(x - 3)$
 - $x^2 + 5x - 84$
= $(x + 12)(x - 7)$
 - $x^2 + 6x - 16$
= $(x + 8)(x - 2)$
 - $x^2 - 3x - 40$
= $(x - 8)(x + 5)$
 - $x^2 - 11x + 18$
= $(x - 9)(x + 2)$
 - $2x^2 - 20x + 32$
= $2(x^2 - 10x + 16)$
= $2(x - 8)(x - 2)$
 - $3x^2 - 24x + 48$
= $3(x^2 - 8x + 16)$
= $3(x - 4)^2$
 - $2x^2 - 2x - 84$
= $2(x^2 - x - 42)$
= $2(x - 7)(x - 6)$
 - $36 - x^2$
= $(6 - x)(6 + x)$
- $4x^2 + 21x + 5$ $4 \times 5 = 20$ (20, 1)
= $4x^2 + 20x + x + 5$
= $4x(x + 5) + 1(x + 5)$
= $(4x + 1)(x + 5)$
 - $3x^2 + 16x + 13$ $3 \times 13 = 39$ (3, 13)
= $3x^2 + 3x + 13x + 13$
= $3x(x + 1) + 13(x + 1)$
= $(3x + 13)(x + 1)$
 - $2x^2 - 2x - 12$ $2 \times 12 = 24$ (4, 6)
= $2x^2 + 4x - 6x - 12$
= $2x(x + 2) - 6(x + 2)$
= $(2x - 6)(x + 2)$ or $2(x - 3)(x + 2)$
 - $8x^2 - 18x + 9$ $8 \times 9 = 72$ (12, 6)
= $8x^2 - 12x - 6x + 9$
= $4x(2x - 3) - 3(2x - 3)$
= $(4x - 3)(2x - 3)$
 - $5x^2 - 13x - 6$ $5 \times 6 = 30$ (10, 3)
= $5x^2 - 10x - 3x - 6$
= $5x(x - 2) + 3(x - 2)$
= $(5x + 3)(x - 2)$
 - $6x^2 + 16x + 10$ $6 \times 10 = 60$ (10, 6)
= $6x^2 + 6x + 10x + 10$
= $6x(x + 1) + 10(x + 1)$
= $(6x + 10)(x + 1)$ or $2(3x + 5)(x + 1)$
 - $6x^2 + 5x - 6$ $6 \times 6 = 36$ (9, 4)
= $6x^2 + 9x - 4x - 6$
= $3x(2x + 3) - 2(2x + 3)$
= $(3x - 2)(2x + 3)$
 - $10x^2 + 6x - 4$ $10 \times 4 = 40$ (10, 4)
= $10x^2 + 10x - 4x - 4$
= $10x(x + 1) - 4(x + 1)$
= $(10x - 4)(x + 1)$ or $2(5x - 2)(x + 1)$
 - $3x^2 - 6x - 24$ $3 \times 24 = 82$ (12, 6)
= $3x^2 - 12x + 6x - 24$
= $3x(x - 4) + 6(x - 4)$
= $(3x + 6)(x - 4)$ or $3(x + 2)(x - 4)$

Quadratic Factoring 2

Factorise all these quadratic expressions!

- $3x^2 + 33x + 84$
= $3(x + 4)(x + 7)$

- $2x^2 + 20x + 32$
= $2(x + 2)(x + 8)$

- $4x^2 + 12x - 72$
= $4(x - 3)(x + 6)$

- $2x^2 + 14x - 36$
= $2(x - 2)(x + 9)$

- $2x^2 - 6x - 80$
= $(x + 5)(x - 8)$

- $3x^2 - 12x - 15$
= $3(x + 1)(x - 5)$

- $2x^2 - 36x + 160$
= $2(x - 8)(x - 10)$

- $5x^2 - 20x + 15$
= $5(x - 1)(x - 3)$

- $4x^2 - 88x + 484$
= $4(x - 11)^2$

- $2x^2 + 20x + 50$
= $2(x + 5)^2$

- $4x^2 - 400$
= $4(x - 10)(x + 10)$

- $6x^2 - 6$
= $6(x + 1)(x - 1)$

- $2x^2 + 5x + 3$
= $(2x + 3)(x + 1)$

- $3x^2 + 8x + 4$
= $(3x + 2)(x + 2)$

- $8x^2 + 14x + 3$
= $(4x + 1)(2x + 3)$

- $3x^2 + 5x - 2$
= $(3x - 1)(x + 2)$

- $2x^2 + 3x - 5$
= $(2x + 5)(x - 1)$

- $5x^2 - 8x + 3$
= $(5x - 3)(x - 1)$

- $3x^2 - 17x + 10$
= $(3x - 2)(x - 5)$

- $2x^2 - 3x - 9$
= $(2x + 3)(x - 3)$

- $2x^2 + 9x - 18$
= $(2x - 3)(x + 6)$

- $15x^2 + x - 6$
= $(5x - 3)(3x + 2)$

- $4x^2 + 2x - 20$
= $2(x - 2)(2x + 5)$

- $8x^2 + 14x + 6$
= $2(4x + 3)(x + 1)$

Quadratic Factoring 3

Factorise all these quadratic expressions!

- $6x^2 + 14x + 4$
= $(2x + 4)(3x + 1)$

- $4x^2 + 6x + 2$
= $(2x + 1)(2x + 2)$

- $8x^2 - 10x + 3$
= $(4x - 3)(2x - 1)$

- $3x^2 + x - 2$
= $(3x - 2)(x + 1)$

- $3x^2 + 16x + 5$
= $(3x + 1)(x + 5)$

- $3x^2 + 11x + 10$
= $(3x + 5)(x + 2)$

- $14x^2 + x - 3$
= $(7x - 3)(2x + 1)$

- $4x^2 - 8x - 5$
= $(2x - 5)(2x + 1)$

- $6x^2 + 13x - 28$
= $(2x + 7)(3x - 4)$

- $2x^2 + 5x + 3$
= $(2x + 3)(x + 1)$

- $5x^2 + 12x + 4$
= $(5x + 2)(x + 2)$

- $4x^2 - 12x - 7$
= $(2x - 7)(2x + 1)$

- $2x^2 - x - 10$
= $(2x - 5)(x + 2)$

- $6x^2 + x - 2$
= $(2x - 1)(3x + 2)$

- $8x^2 + 14x + 3$
= $(4x + 1)(2x + 3)$

- $2x^2 + 9x - 5$
= $(2x - 1)(x + 5)$

- $2x^2 - 24x + 40$
= $(2x - 4)(x - 10)$

- $16x^2 - 49$
= $(4x + 7)(4x - 7)$

- $2x^2 - 50$
= $2(x + 5)(x - 5)$

- $3x^2 - 147 = 3(x^2 - 49)$
= $3(x - 7)(x + 7)$

- $4 - 36x^2$
= $4(1 - 3x)(1 + 3x)$

- $2x^2 + 4x - 48 = 0$
 $(2x - 8)(x + 6) = 0$

$x = 4$ or $x = -6$

$2x^2 + 2x - 12 = 0$

$(2x + 6)(x - 2) = 0$

$x = -3$ or $x = 2$

$2 \times -3 = -6$ and $2 \times 2 = 4$

The solutions are 2x bigger

Algebra (1)

Simplify all these expressions

- $3a + 8a = 11a$
- $b + b + 5b = 7b$
- $4c + 6c - 8c = 2c$
- $5d - d - 3e - 2e = 4d - 5e$
- $5e + 2f + 3e - 4f = 8e - 2f$
- $3g - 3h + 4g - 5h = 7g - 8h$
- $a \times a \times 5 = 5a^2$
- $9b \times b \times b \times b = 9b^4$
- $14y \times 2 = 28y$
- $c \div c = 1$
- $12d \div 4 = 3d$
- $5e \times 6 = 30e$
- $4f \times f = 4f^2$
- $2g \times 8h = 16gh$
- $20m \div 10m = 2$
- $15xy \div 3 = 5xy$
- $6r \times 7r = 42r^2$
- $5g^2 \times 4g = 20g^3$

Expand and simplify all these expressions

- $2(a + 4b) = 2a + 8b$
 - $5(3c - 8) = 15c - 40$
 - $-10(2d - 3) = -20d + 30$
 - $-2(4f + 3e) = -8f - 6e$
 - $2(a + 2) + (a - 1)$
 $= 3a + 3$
 - $4(b - 1) + 2(b + 2)$
 $= 6b + 4$
 - $(c - d) + 6(c + 2d)$
 $= 7c + 11d$
 - $3(e + 9) - 2e$
 $= e + 27$
 - $f(2f - 4) + 4f$
 $= 2f^2$
-
- $x = 3$ and $y = 4$
Find the values:
-
- $2x + 3y = 18$
 - $2(x + y) = 14$
 - $5x - 2y = 7$
 - $x^2 - y^2 = -7$
 - $3x^2 = 27$
 - $\frac{6x + y}{2} = 4$
 - $xy^2 = 48$
 - $\sqrt{(4x + y)} = 4$

Algebra (2)

Simplify all these expressions

$$3a + 2a - 4a = 2a$$

$$6b - 4b - 3b = -b$$

$$2c - 3d + c - d = 3c - 4d$$

$$ab + ba = 2ab$$

$$a^2 \times a \times 5 = 5a^3$$

$$9b \times 6b = 54b^2$$

$$14y \times 2 = 28y$$

$$c^2 \div c = c$$

$$d^3 \div d^2 = d$$

$$5e \times 8e = 40e^2$$

$$4f \times 9g = 36fg$$

$$2g \times 8g^2 = 16g^3$$

$$8m^3 \div 2m^2 = 4m$$

$$15x^3 \div 3x^3 = 45$$

$$4d^2 + 4d^2 = 8d^2$$

$$ef + 5ef - 4ef = 2ef$$

$$4ab \times 4ac \div 4bc = 4a^2$$

$$(3x^2y^3)^2 = 9x^4y^6$$

$$\sqrt{(25r^2s^6)} = 5rs^3$$

$$a^3 \times a^5 = a^8$$

$$(b^5)^2 = b^{10}$$

$$c^5 \div c^3 = c^2$$

$$d^7 \times d^7 = d^{14}$$

$$(3e^4)^3 = 27e^{12}$$

$$f \times f^3 \times f^3 = f^7$$

$$3g^5 \times 2g^3 = 6g^8$$

$$3h^6 \div h^2 = 3h^4$$

$$2(3a - 5) = 6a - 10$$

$$4(6b + 8c) = 24b + 32c$$

$$5d(d^2 - d + 2) = 5d^3 - 5d^2 + 10d$$

$$5a + (-2a) = 3a$$

$$b - (-2b) = 3b$$

$$(-5c) - (-8c) = 3c$$

$$(-2d) + (-2d) = 0$$

$$-8y \times 4y = -32y^2$$

$$6 \times -4r^2 = -24r^2$$

$$-3g^2 \times -4g^3 = 12g^5$$

$$-6x \div -6x = 1$$

Algebra (3)

Simplify all these expressions

- $13a + (2a - 8a) = 7a$

- $2b - 3c + b - 2c = 3b - 5c$

- $6a^2 \times 7 \times a = 42a^3$

- $9b \times 6b^3 = 54b^4$

- $14y \times 2y = 28y^2$

- $c^8 \div c^2 = c^6$

- $(8d)^2 = 64d^2$

- $e^3 + e^3 = 2e^3$

- $4f^2 + 4f^2 = 8f^2$

- $5ab \times 5ac \div 5bc = 5a^2$

- $(8e^2f^3)^2 = 64e^4f^6$

- $\sqrt{(16g^2h^8)} = 4gh^4$

- $(-2x)^2 = 4x^2$

- $-(3x)^3 = -27x^3$

- $2(a + 2b) + (a - b)$

$$= 3a + 5b$$

- $x(2x + 3) - 4(3x - 1)$

$$= 2x^2 - 9x + 4$$

- $x(x^2 + 1) - x^2(x + 1)$

$$= x - x^2$$

- $\frac{12a^2b}{3ab} = 4a$

- $\frac{6cd^2}{2c^2d} = \frac{3d}{c}$

- $\frac{10e}{3} - \frac{2e}{9} = \frac{28e}{9}$

- $\frac{d}{21} + \frac{4d}{7} = \frac{13d}{21}$

- $\frac{3x}{8} - \frac{x}{6} = \frac{10x}{48}$

- $\frac{5a}{2} + \frac{a}{10} = \frac{26a}{10}$

- $\frac{4c}{15d} \times \frac{5e}{8} = \frac{ce}{6d}$

- $\frac{3}{4e} \div \frac{15}{8e^2} = \frac{2e}{5}$

- $\frac{3ab}{10cd} \times \frac{2c^3}{9b^2} = \frac{ac^2}{15db}$

- $\frac{8}{9x} \div \frac{2}{3x} = \frac{4}{3}$

- $\frac{a+1}{2} + \frac{a-1}{3} = \frac{5a+1}{6}$

- $\frac{b+c}{2} - \frac{b-2}{5} = \frac{3a+5c+4}{10}$

- $\frac{3x-1}{3} + \frac{2-x}{4} = \frac{9x+2}{12}$

- $\frac{4a+1}{2} - \frac{a+3}{5} = \frac{18a-1}{10}$

$$x = 5 \quad y = 3 \quad z = 1$$

- $6x + 2y = \underline{\quad 36 \quad}$

- $x^2 + y^2 = \underline{\quad 34 \quad}$

- $2x^2 + 3z = \underline{\quad 53 \quad}$

- $(4x - 2z)^2 = \underline{\quad 324 \quad}$

$$a = \frac{1}{2} \quad b = \frac{1}{8} \quad c = \frac{3}{4}$$

- $a + b = \underline{\quad \frac{5}{8} \quad}$

- $24a^2 + 4c = \underline{\quad 9 \quad}$

- $\frac{a}{b} = \underline{\quad 4 \quad}$

- $4c(12a^2b) = \underline{\quad \frac{9}{8} (1 \frac{1}{8}) \quad}$

Equations (1)

Solve all these equations!

- | | | | |
|-----------------------|----------|--------------------------------|----------|
| ▶ $4x = 44$ | $x = 11$ | ▶ $3(x + 2) - x = 30$ | $x = 12$ |
| ▶ $x - 8 = 9$ | $x = 17$ | ▶ $4(1 + 2x) = 52$ | $x = 6$ |
| ▶ $\frac{1}{4}x = 8$ | $x = 32$ | ▶ $6(x + 1) - 6 = 30$ | $x = 5$ |
| ▶ $x + 7 = 19$ | $x = 12$ | ▶ $2(x - 9) = x - 4$ | $x = 14$ |
| ▶ $6x = 48$ | $x = 8$ | ▶ $8x = 2(3x + 6)$ | $x = 6$ |
| ▶ $\frac{1}{2}x = 20$ | $x = 40$ | ▶ $5(x + 8) = 50$ | $x = 2$ |
| ▶ $x - 5 = 25$ | $x = 30$ | ▶ $(x + 1) + 2(x + 2) = 29$ | $x = 8$ |
| ▶ $5x = 0$ | $x = 0$ | ▶ $4(3x - 1) + 10 = 18$ | $x = 1$ |
| ▶ $x - 14 = 5$ | $x = 19$ | ▶ $x + (x + 2) + (x + 4) = 21$ | $x = 5$ |
| ▶ $x + 8 = 24$ | $x = 16$ | ▶ $6x - 3 = 2x + 25$ | $x = 7$ |
| ▶ $3x + 2 = 14$ | $x = 4$ | ▶ $4(x + 5) = 3(16 - x)$ | $x = 4$ |
| ▶ $14 + 4x = 22$ | $x = 2$ | ▶ $8x - 5 = 2x + 13$ | $x = 3$ |
| ▶ $12x - 5 = 31$ | $x = 3$ | ▶ $6x - 4 = 2(x + 14)$ | $x = 8$ |
| ▶ $8x - 21 = 11$ | $x = 4$ | | |
| ▶ $3x = x + 10$ | $x = 5$ | | |
| ▶ $7x + 3 = 33 + x$ | $x = 5$ | | |
| ▶ $15x - 4 = 3x + 8$ | $x = 1$ | | |
| ▶ $4 + 6x = 12 + 2x$ | $x = 2$ | | |

Equations (2)

Solve all these equations!

- | | | | |
|-------------------------------|-----------|-------------------------------|-----------|
| ■ $a - 5 = 19$ | $a = 24$ | ■ $2(2a - 5) + 4(a + 7) = 82$ | |
| ■ $b + 19 = 38$ | $b = 19$ | | $b = 8$ |
| ■ $7c = 91$ | $c = 13$ | ■ $5(c + 3) + (c - 6) = 27$ | |
| ■ $\frac{d}{5} = 20$ | $d = 100$ | | $c = 3$ |
| ■ $11e - 5 = 72$ | $e = 7$ | ■ $2(e + 5) - 3(6 - e) = 42$ | |
| ■ $2f + 4 = 80$ | $f = 38$ | | $e = 10$ |
| ■ $g + 5g = 72$ | $g = 12$ | ■ $5(f - 1) + 3(f - 4) = 23$ | |
| ■ $3h = 90 - 2h$ | $h = 18$ | | $f = 5$ |
| ■ $3j + 2 = 27 - 2j$ | $j = 5$ | ■ $22 - 5k - (k + 10) = 0$ | |
| ■ $36 - 4k = 0$ | $k = 9$ | | $k = 2$ |
| ■ $12m - 5 = 15 + 8m$ | | ■ $2(3m - 4) - 2(m + 7) = 0$ | |
| | $m = 5$ | | $m = 5.5$ |
| ■ $2n + 7 = 31 - 4n$ | | ■ $5n - 2 = n - 3(n + 3)$ | |
| | $n = 4$ | | $n = -1$ |
| ■ $3(p + 2) - p = 26$ | | | |
| | $p = 10$ | | |
| ■ $4(2r - 6) + 3(r + 5) = 35$ | | | |
| | $r = 4$ | | |
| ■ $\frac{1}{2}(t - 4) = 12$ | | | |
| | $t = 28$ | | |
| ■ $\frac{w + 7}{5} = 5$ | | | |
| | $w = 18$ | | |

Equations (3)

Solve all these equations!

$$\blacksquare 3a + 5 = 17 - a$$

$$a = 3$$

$$\blacksquare 25 - 7c = -10$$

$$c = 5$$

$$\blacksquare 12d - 5 = 15 + 10d$$

$$d = 10$$

$$\blacksquare 2e + 7 = 31 - 4e$$

$$e = 4$$

$$\blacksquare 3(f + 2) - f = 22$$

$$f = 8$$

$$\blacksquare \frac{1}{4}(g - 12) = 9$$

$$g = 48$$

$$\blacksquare \frac{h+7}{4} = 10$$

$$h = 33$$

$$\blacksquare \sqrt[3]{j} - 2 = 10$$

$$j = 16$$

$$\blacksquare 4k - 5 = 2k + 43$$

$$k = 24$$

$$\blacksquare 5(m + 3) + (m - 5) = 100$$

$$m = 15$$

$$\blacksquare 3(2n - 5) - 4(n + 7) = 13$$

$$n = 28$$

$$\blacksquare \frac{2a-5}{3} = \frac{a-2}{2}$$

$$a = 4$$

$$\blacksquare \frac{c-1}{3} = c - \frac{3(c+2)}{5}$$

$$c = 13$$

$$\blacksquare \sqrt[3]{4}(5e - 12) - \frac{1}{4}(4 + e) = 32$$

$$e = 12$$

$$\blacksquare x^2 = 81$$

$$x = \pm 9$$

$$\blacksquare x^2 = 0.36$$

$$x = \pm 0.6$$

$$\blacksquare 3^x = 81$$

$$x = 4$$

$$\blacksquare x^3 = 64$$

$$x = 4$$

$$\blacksquare \sqrt[3]{x} = 2$$

$$x = 8$$

$$\blacksquare 4^x = \frac{1}{16}$$

$$x = -2$$

$$\blacksquare \text{Let } x = 2y.$$

$$\text{Find the value of } \frac{x^2 - 2y^2}{x^2 + 2y^2}$$

$$= \frac{1}{3}$$

±

Equations (4)

Solve all these equations

- $3^{x+1} = 81$

$$x = 3$$

- $2^{x-1} = 64$

$$x = 7$$

- $5 \times 5^{3x} = 5^{-2x^2}$

$$x = -1 \text{ or } x = -\frac{1}{2}$$

- $7 \times 7^{1-x} = 7^{3x^2}$

$$x = -1 \text{ or } x = -\frac{1}{2}$$

- $2^{3x+4} > 2^x$

$$x = \frac{2}{3} \text{ or } x = -1$$

- $5^{x+6} = 5^{x^2}$

$$x = 3 \text{ or } x = 2$$

- $9 \times 3^{x-4} < 27$

$$x < 5$$

- $10 \times 2^{p-1} < 1280$

$$p < 8$$

- $25 \times 5^{(2y+13)} = 5^{y^2}$

$$y = 5 \text{ or } y = -3$$

- $9 \times 3^{(x-4)} > 81$

$$x > 6$$

- $216 \times 6^{(2x+5)} = 6^{x^2}$

$$x = 4, x = -2$$

- $8 \times 2^{(2y+13)} = 2^y$

$$y = -16$$

- $64 \times 4^{x-5} > 512$

$$x > \frac{13}{2} \text{ (6.5)}$$

- $16 \times 2^x = 2^{6x+3}$

$$x = \frac{1}{5}$$

- $6 \times 2^{2n+1} > 96$

$$n > \frac{3}{2}$$

- $9 \times 3^x = 3^{5x+4}$

$$x = -\frac{1}{2}$$

Simultaneous Equations

Solve all these equations on your own paper!

1. $y = 2x + 4$
 $y = 5x - 5$
 $x=3, y=10$

2. $x = -y + 12$
 $x = 2y$
 $x=8, y=4$

3. $y = 7$
 $y = 3x - 20$
 $x=9, y=7$

4. $x = 1 - 2y$
 $x = y - 5$
 $x=-3, y=2$

5. $x = 2$
 $x = 3y - 1$
 $x=2, y=1$

6. $y = 6x - 1$
 $y = 10x - 1$
 $x=0, y=-1$

7. $y = 3x + 1$
 $y = 2x - 2$
 $x=-3, y=-8$

8. $x = 3y$
 $x = 5 - 2y$
 $x=3, y=1$

9. $y = 10$
 $y = 2 - 4x$
 $x=-2, y=10$

10. $x = y - 4$
 $x = 3 - 6y$
 $x=-3, y=1$

11. $y = 5x - 6$
 $2x + y = 8$
 $x=2, y=4$

12. $x = 3y + 6$
 $2y + x = 1$
 $x=3, y=-1$

13. $x = 5 - 3y$
 $13 = x + 2y$
 $x=29, y=-8$

14. $x = 13 - 2y$
 $3y = x + 27$
 $x=-3, y=8$

15. $y = 6x - 5$
 $y + 2x = -5$
 $x=0, y=-5$

16. $x + y = 3$
 $2x - 3 = y$
 $x=8.3, y=2.3$

17. $2x + y = 7$
 $y = 4x - 5$
 $x=2, y=3$

18. $5y - x = 2$
 $x = 4y - 1$
 $x=3, y=1$

19. $5 = x + 8y$
 $x = 7 - 10y$
 $x=-3, y=1$

20. $x = 3 - 3y$
 $x = 2y + 18$
 $x=12, y=-3$

21. $x + y = 14$
 $x - y = 10$
 $x=12, y=2$

22. $-x + y = 2$
 $x + y = 28$
 $x=13, y=15$

23. $x - 2y = 4$
 $x + 2y = 14$
 $x=9, y=2.5$

24. $-3x + y = -5$
 $3x + y = 19$
 $x=4, y=7$

25. $y + 2x = 1$
 $4x - y = -25$
 $x=-4, y=9$

26. $4y - x = -5$
 $x + 3y = 5$
 $x=5/4, y=-15/16$

27. $x + 2y = 1$
 $x - 2y = -11$
 $x=-5, y=3$

28. $5x + y = 17$
 $-5x + y = -3$
 $x=2, y=7$

29. $-y + 3x = 8$
 $3x + y = 16$
 $x=4, y=4$

30. $7x + y = 11$
 $y + 3x = 3$
 $x=2, y=-3$



Simultaneous Equations

1. $x = \text{apples}; y = \text{oranges}$

$$x + y = 8$$

$$3x + 2y = 20$$

Solution: 4 apples and 4 oranges

2. $x = \text{muffins}; y = \text{cupcakes}$

$$x + y = 27$$

$$6x + 4.5y = 117$$

Solution: 12 muffins and 10 cupcakes.

3. $x = \text{paperback}; y = \text{hardcover books.}$

$$x + y = 12$$

$$30x + 45y = 960$$

Solution: 20 paperbacks and 8 hardcovers.

4. $x = \text{pens}; y = \text{pencils}$

$$x + y = 18$$

$$12x + 8y = 224$$

Solution: 12 pens and 10 pencils.

5. $x = \text{sopranos}; y = \text{altos}$

$$x + y = 28$$

$$4x + 3y = 101$$

Solution: 13 sopranos and 15 altos.

6. $x = \text{red marbles}; y = \text{blue marbles.}$

$$x + y = 40$$

$$8x + 3y = 225$$

Solution: 20 red marbles and 20 blue marbles.

7. $x = \text{boys}; y = \text{girls}$

$$x + y = 25$$

$$20x + 25y = 550$$

Solution: 10 girls, 15 boys.

Simultaneous Equations

8. Solution: 700 Adults, 300 Children
9. Solution: $x = 5\% \$10,000$, $y = 8\% \$20,000$
10. $x =$ present age of the woman; $y =$ the present age of her son
 Present age of the mother in relation to her son: $x = y + 30$
 15 years ago: $x - 15 = 2(y - 15)$.
 Solution: y (son) = 45 years, x , (mother), is 75 years.
11. $x =$ the number of adult tickets; $y =$ the number of children's tickets.
 Solution: $x = 500$, $y = 225$
12. $x =$ amount invested at 6%; $y =$ amount invested at 7%
 Total investment: $x + y = \$20,000$
 Total interest: $0.06x + 0.07y = \$1,280$
 Solution: $x = \$12,000$. $y = \$8,000$.
13. 9 litres of 30% alcohol solution $\rightarrow 0.3 \times 9 = 2.7$ litres of pure alcohol
 $x =$ number of litres 20% solution; $y =$ number of litres 50% solution.
 Total number of litres: $x + y = 9$
 Total litres of pure alcohol: $0.2x + 0.5y = 2.7$
 Solution: $x = 6$ litres $y = 3$ litres.
14. Let $x =$ speed of the boat in still water; $y =$ the speed of the current.
 Downstream speed: $x + y = 24$
 Upstream speed: $x - y = 18$
 Solution: $x = 9$ kmph. $y = 3$ kmph.
15. Let $x =$ the amount of money Andre has, $y =$ amount Bob has.
 Andre gives Bob \$20, they have the same amount $x - 20 = y + 20$
 Bob gives Andre \$22, Andre has twice as much as Bob $x + 22 = 2(y - 22)$
 Solution: Bob has \$106. Andre has \$146.

Changing the Subject

Make x the subject of the equations

$$\blacktriangleright 2x + 5y = 15$$

$$x = \frac{15 - 5y}{2}$$

$$\blacktriangleright 4y = 2x + 8$$

$$x = 2y - 4$$

$$\blacktriangleright 3x - y = 12$$

$$x = 4 + \frac{y}{3}$$

$$\blacktriangleright xy = 19$$

$$x = \frac{19}{y}$$

$$\blacktriangleright ax = \frac{b}{10}$$

$$x = \frac{b}{10a}$$

$$\blacktriangleright \frac{3}{x} = \frac{4}{2}$$

$$x = \frac{6}{y}$$

$$\blacktriangleright \sqrt{\frac{x}{5}} = y$$

$$x = 5y^2$$

$$\blacktriangleright 2x^2 = 3y$$

$$x = \sqrt{\frac{3y}{2}}$$

$$\blacktriangleright 12y = \frac{x}{6}$$

$$x = 72y$$

$$\blacktriangleright x^2 = 5y$$

$$x = \sqrt{5y}$$

$$\blacktriangleright y^2 = x^2 + 2a$$

$$x = \sqrt{y^2 - 2a}$$

$$\blacktriangleright T = \frac{1}{4}\pi x^2$$

$$x = \sqrt{\frac{4T}{\pi}}$$

$$\blacktriangleright \frac{3}{8}kx = y$$

$$x = \frac{8y}{3k}$$

$$\blacktriangleright \frac{x}{11} = \frac{y}{5}$$

$$x = \frac{11y}{5}$$

$$\blacktriangleright x^2 + y^2 = r^2$$

$$x = \sqrt{r^2 - y^2}$$

$$\blacktriangleright s = 4.9x^2$$

$$x = \sqrt{\frac{s}{4.9}}$$

Simultaneous Equations

Solve all these simultaneous equations!

$$\begin{aligned} \blacktriangleright \quad & x + y = 10 \\ & x - y = 12 \\ & \quad \quad x = 11, y = -1 \end{aligned}$$

$$\begin{aligned} \blacktriangleright \quad & 5x + y = 14 \\ & 4x - y = 22 \\ & \quad \quad x = 4, y = -6 \end{aligned}$$

$$\begin{aligned} \blacktriangleright \quad & 2x + 3y = -2 \\ & 3x - y = 13 \\ & \quad \quad x = -69, y = 47 \end{aligned}$$

$$\begin{aligned} \blacktriangleright \quad & -4x + 5y = 20 \\ & 4x - 3y = -4 \\ & \quad \quad x = 5, y = 8 \end{aligned}$$

$$\begin{aligned} \blacktriangleright \quad & 5x + 8y = 71 \\ & 7x - 3y = 71 \\ & \quad \quad x = 11, y = 8 \end{aligned}$$

$$\begin{aligned} \blacktriangleright \quad & 3x + 5y = 5x - 3y \\ & 3x + 5y = 19 - 2y \\ & \quad \quad x = 4, y = 1 \end{aligned}$$

$$\begin{aligned} \blacktriangleright \quad & 2x + y = 13 \\ & x = 8 - y \\ & \quad \quad x = 5, y = 3 \end{aligned}$$

$$\begin{aligned} \blacktriangleright \quad & x = 4y - 1 \\ & 3x + 2y = 11 \\ & \quad \quad x = 3, y = 1 \end{aligned}$$

$$\begin{aligned} \blacktriangleright \quad & 2x = 20 + 2y \\ & 4x + y = 20 \\ & \quad \quad x = 6, y = -4 \end{aligned}$$

$$\begin{aligned} \blacktriangleright \quad & y = 6x - 5 \\ & y + 2x = -5 \\ & \quad \quad x = 0, y = -5 \end{aligned}$$

$$\begin{aligned} \blacktriangleright \quad & y = 5x - 2 \\ & y = 4x + 4 \\ & \quad \quad x = 6, y = 28 \end{aligned}$$

$$\begin{aligned} \blacktriangleright \quad & -2x + 3y = 3 \\ & 7x + 2y = 27 \\ & \quad \quad x = 3, y = 3 \end{aligned}$$

Sequences

Use $n = 1, 2, 3$ and 4 to find the first 4 terms of each sequence

- $3n - 1$

n	1	2	3	4
	2	5	8	11

- $100 - 10n$

n	1	2	3	4
	90	80	70	60

- $5n + 4$

n	1	2	3	4
	9	14	19	24

- $-2n + 3$

n	1	2	3	4
	1	-1	-3	-5

- 10^n

n	1	2	3	4
	10	100	1000	10000

- 2^n

n	1	2	3	4
	2	4	8	16

- $n(n + 3)$

n	1	2	3	4
	4	10	18	28

- $n^2 + n$

n	1	2	3	4
	2	6	12	20

- $6n + 4$

n	1	2	3	4
	10	16	22	28

- $25 - 3n$

n	1	2	3	4
	22	19	16	13

- $2n^2 - n$

n	1	2	3	4
	1	6	15	28

- $n + 2^n$

n	1	2	3	4
	3	6	11	20

- $n^3 - 1$

n	1	2	3	4
	0	7	26	63

- $n^2(n - 2)$

n	1	2	3	4
	-1	0	9	32

- $\frac{n}{n+2}$

n	1	2	3	4
	0.33	0.5	0.6	0.67

- $\frac{n}{2n+1}$

n	1	2	3	4
	0.33	0.4	0.43	0.44

Sequences

Find the next 2 terms and the rule for all these sequences!

■ 3, 7, 11, 15, 19, 23, 27

$$t_n = 4n - 1$$

■ 6, 9, 12, 15, 18, 21, 24

$$t_n = 3n + 3$$

■ -3, -1, 1, 3, 5, 7, 9

$$t_n = 2n - 5$$

■ 6, 11, 16, 21, 26, 31, 36

$$t_n = 5n + 1$$

■ 4, 2, 0, -2, -4, -6, -8

$$t_n = -2n + 6$$

■ 95, 90, 85, 80, 75, 70, 65

$$t_n = -5n + 100$$

■ 7, 4, 1, -2, -5, -8, -11

$$t_n = -3n + 10$$

■ 6.5, 7, 7.5, 8, 8.5, 9, 9.5

$$t_n = 0.5n + 6$$

■ 1, 4, 9, 16, 25, 36, 49

$$t_n = n^2$$

■ 3, 6, 11, 18, 27, 38, 51

$$t_n = n^2 + 2$$

■ 0, 3, 8, 15, 24, 35, 48

$$t_n = n^2 - 1$$

■ 3, 10, 21, 36, 55, 78, 105

$$t_n = 2n^2 + n$$

■ 3, 8, 15, 24, 35, 48, 63

$$t_n = n^2 + 2n$$

■ 2, 10, 24, 44, 70, 102, 140

$$t_n = 3n^2 - n$$

■ 6, 12, 20, 30, 42, 56, 72

$$t_n = n^2 + 3n + 2$$

■ 8, 14, 24, 38, 56, 78, 104

$$t_n = 2n^2 + 6$$

Sequences

Can you answer these sequence questions?

- The first five terms in a number sequence are: 7, 10, 13, 16, 19
 (a) n th term of the sequence is $t_n = 3n + 4$
 (b) 100th term in this number sequence $3 \times 100 + 4 = 304$.
- A number sequence has n th term $6n + 5$
 (a) First four terms of this sequence are 11, 17, 23, 29
 (b) Not correct $(1008 - 5) / 6 = 167.2$ (needs to be an integer)
- The first 5 terms of a number sequence are: 1, 5, 9, 13, 17.
 (a) n th term of the sequence is $t_n = 4n - 3$
 (b) Is 97 a term in this sequence? $4 \times 25 - 3 = 97$ yes 25th term.
- The n th term of a number sequence is $n^2 + 5$.
 (a) First three terms of this sequence are 6, 9, 14
 (b) $t_{10} = 105$, $t_5 = 30$, difference = 75
- The first 5 terms in a number sequence are: 13, 10, 7, 4, 1.
 (a) n th term of the sequence is $t_n = -3n + 16$
 (b) 50th term of the sequence is $-3 \times 50 + 16 = -134$
- Kim calculates first 50 terms of the sequence $150 - 4n$.
 First negative term $150 - 4n = 0$, $n = 37.5$, term 38 = -2
- The n th term of a sequence is $(n + 2)(n + 5)$
 First 5 terms of the sequence: 18, 28, 40, 54, 70.
- The n th term of a sequence is $5n - 9$
 (a) First 5 terms are: -4, 1, 6, 11, 16
 (b) Term 99 = 486, term 100th = 491, difference = 5
 The last term of this sequence is 741.
 (c) How many terms in the sequence? $5 \times 150 - 9 = 150$ terms

Sequences

Can you answer these harder sequence questions?

1. A linear sequence starts 300, 296, 292, 288 ...

Which term will be the first to have a negative value?

$$t_n = -4n + 304 \quad 4n = 304 \quad \text{first negative term } n = 76\text{th term}$$

2. The n th term of a sequence is $4n - 10$

Write an expression for the term $n + 1$

first 5 terms for $4n - 10 = -6, -2, 2, 6, 10,$

first 5 terms for $n+1 = -2, 2, 6, 10, 14, t_n = 4n - 6$

3. A linear sequence starts off 4, 11, 18, 25, ...

Write an expression for the n th term.

How many terms in the sequence will be less than 150?

$$T_n = 7n - 3, T_{22} = 151 \text{ so } 21 \text{ terms } < 150$$

4. n th term of Sequence A: 4, 7, 10, 13, $t_n = 3n + 1$

n th term of Sequence B: 16, 49, 100, 169, $T_n = 9n^2 + 6n + 1$

Show that the 30th term of sequence B equals 49×169 .

$$49 \times 169 = 8281 \quad T_{30} = 9(30)^2 + 6(30) + 1 \\ = 8281$$

5. A linear sequence has the terms $a+b, a+3b, a+5b, a+7b, a+9b, \dots$

Find the n th term of the sequence.

$$\text{Term 8: } a + 15b = 59$$

$$\text{Term 5: } a + 9b = 35$$

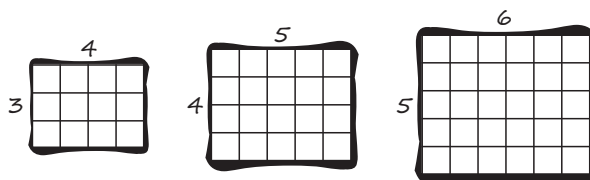
$$6b = 24, \quad b = 4, \quad a = -1$$

Sequence is therefore: 3, 11, 19, 27, 35, $T_n = 8n - 5$

6. A pattern of rectangles continues as below. The areas of each rectangle form a sequence: 12, 20, 30, 42, ...

Find the n th term of the sequence of areas.

$$t_n = n^2 + 5n + 6$$



Algebra Practise

Basic Skill Questions - Set A

1. $(x + 2)(4x - 5) = 4x^2 + 3x - 10$

2. $2x^2 - 15x + 18 = (2x - 3)(x - 6)$

3. Coordinates cut x axis at $y = 0$
 $= (0, 0)(-3, 0)$

4. If $x = 2$ and $h = x(x - 1) + 2$
 $2(2 - 1) + 2, h = 4$

5. $(3x + 7)(x - 2) = 3x^2 + x - 14$

6. $3x^2 - 11x + 6 = (3x - 2)(x - 3)$

7. If $x = 2$ and $h = 2x(x - 1.5) + 1$
 $2(2)(2 - 1.5) + 1, h = 3$

8. Coordinates = $(0, 0), (-9, 0)$

9. $4(x + 5) = 24, x = 1$

10. $(x + 7) \div 4 = 5, x = 13$

11. Area = $w(w + 2)$
 $= w^2 + 2w$

12. Area = $L(L - 3)$
 $= L^2 - 3L$

▲ 13. $x^2 - x - 2$

▼ $= (x + 1)(x - 2)$

▲ 14. $x^2 + 5x - 36.$

▼ Length of each side = $(x - 4)(x + 9)$

▼ 15. $y = 3x^2 - 2x + 5$

▲ $y = 3(4)^2 - 2(4) + 5$

▼ $y = 45$

▼ 16. $y = 3x^2 - 5x + 7$

▲ $y = 3(2)^2 - 5(2) + 7$

▼ $y = 9$

▲ 17. $n^2 - 4n - 5.$

▼ $= (n + 1)(n - 5)$

▼ 18. $x^2 + 4x - 12$

▲ $= (x + 6)(x - 2)$

▼ length = $x+2$ width = $x-2$

▼ 19. $y = -(x^2 - 4x)$

▼ x measures the horizontal distance travelled

▼ 20. $d = ut + 3t^2$

▼ $d = (3 \times 5) + 3(5)^2$

▲ $d = 90$

Algebra Practise

Basic Skill Questions - Set B

1. $A = xy + 5y^2$

$$A = 2 \times 4 + 5 \times 4^2$$

$$A = 88$$

2. $3x^2 - 4x - 32$

$$(3x + 8)(x - 4)$$

3. $h = 9 - 4x^2$ $4x^2 = 9 - h$

$$x^2 = \frac{9 - h}{4}$$

$$x = \sqrt{\frac{9 - h}{4}}$$

4. $n = 9m^2 - 16$

$$9m^2 = n + 16$$

$$m^2 = \frac{n + 16}{9}$$

$$m = \sqrt{\frac{n + 16}{9}}$$

5. $3x^2 + 2x - 40$

$$= (3x - 10)(x + 4)$$

6. $(2x + 3)(x - 2)$

$$= 2x^2 - x - 6$$

7. $3^4 = 81, x = 3$

8. $2^6 = 64, x = 7$

9. $r^2 = \frac{9A}{25}$ $r = \sqrt{\frac{9A}{25}}$

10. $r^2 = \frac{4A}{9}$ $r = \sqrt{\frac{4A}{9}}$

▲ 11. $A = 3(2x^2y - 2x)$

▼ $A = 3(2 \times 3^2 \times 5 - 2 \times 3)$

▲ $A = 252$

▼ 12. $A = 2(x + 3xy^2)$

▼ $A = 2(5 + 3 \times 5 \times 2^2)$

▲ $A = 130$

▼ 13. $(3x - 2)(x + 2)$

▼ $= 3x^2 + 4x - 4$

▲ 14. $2x^4 - 3x + 5$

▼ $= 2(-2)^4 - 3(-2) + 5$

▲ $= 43$

▼ 15. $60 \div x + 12 = 14$

▼ $x = 30$

▲ 16. $w = pq^2 + r$

▲ $p = \frac{w - r}{q^2}$

▼ 17. $5m^2n \times m^3n^2 = 5m^5n^3$

▼ 18. a. $4a^3 = 32$ $a = 2$

▲ b. $(b^3)^2 = 64$ $b = 2$

▼ c. $5^c = 125$ $c = 3$

▼ d. $(2d^2)^2 = 324$ $d = 3$

▼ 19. $12a^2b + 6ab^2 - 7a^2b$

▲ $= 5a^2b + 6ab^2$

▼ 20. $3(2x + 7) = 9$

▲ $x = -2$

Algebra Practise

Proficiency Skill Questions - Set A

1. $h = \text{hours}$ $8 + 3(h - 2) = 23$

$$8 + 3h - 6 = 23$$

$$3h + 2 = 23$$

$$h = 7$$

James hires the bike for 7 hours.

2. $10 \times 2^{n-1} \geq 160$

$$2^{n-1} \geq 16$$

$$2^4 = 16$$

$$\text{therefore } n - 1 \geq 4$$

$$n \geq 5$$

3. $5r^3 = 320$

$$r^3 = 64$$

$$r = 4$$

4. $\text{area} = x^2 - x - 2.$

Factorise $(x + 1)(x - 2)$

Width = $(x - 2)$

5. Show that $\frac{2}{x} + \frac{3+x}{5} = \frac{x^2 + 3x + 10}{5x}$

$$\frac{2}{x} + \frac{3+x}{5} = \frac{5 \times 2 + 3x + x^2}{5x}$$

$$= \frac{x^2 + 3x + 10}{5x}$$

6. $= \frac{3ab^2}{4ab^2} - \frac{4a^3b}{4ab^2} + \frac{ab^2}{4ab^2}$

$$= \frac{3}{4} - \frac{a^2}{b} + \frac{1}{4}$$

$$= 1 - \frac{a^2}{b}$$

7. $x^2 + 3x - 10 = (x + 5)(x - 2)$

$$y = 0 \text{ when } x = -5 \text{ or } x = 2$$

calculating when $y < 0$

test between $x = -5$ and $x = 2$

$$x = 0, y = 0^2 + 3 \times 0 - 10$$

$y = -10$, therefore y is negative

when $-5 < x < 2$

8. $x^2 + 4x - 12 = (x + 6)(x - 2)$

$$y = 0 \text{ when } x = -6 \text{ or } x = 2$$

$$\text{test } x = 3, y = 3^2 + 4 \times 3 - 12$$

$y = 9$, therefore y is positive when

$x < -6$ or $x > 2$

9. $D = 7 \times 2^{n-1}, 112 = 7 \times 2^{n-1}$

$$16 = 2^{n-1}, 2^4 = 16$$

$$n = 5 \text{ (5 weeks)}$$

10. $77 = 38 + 13h$

$$39 = 13h, h = 3 \text{ hours}$$

11. $= \frac{5xy^2}{4xy^2} - \frac{2a^3y}{4xy^2} + \frac{xy^2}{4xy^2}$

$$= \frac{5}{4} - \frac{x^2}{2y} + \frac{1}{4} \qquad \frac{5}{4} + \frac{1}{4} = \frac{3}{2}$$

$$= \frac{3}{2} - \frac{x^2}{2y}$$

$$= \frac{3y - x^2}{2y}$$

12. $486 = 6r^4$

$$81 = r^4$$

$$r = 3$$

rate of spread is $3m^2$ per week

Algebra Practise

Proficiency Skill Questions - Set B

1. $x^2 + 2x - 8 = 0$

$$(x + 4)(x - 2) = 0, x = -4 \text{ or } x = 2$$

2. $x^2 + 4x - 12$

$$= (x + 6)(x - 2) \rightarrow \text{side lengths}$$

$$x^2 + 4x - 12 = 128$$

$$x^2 + 4x - 140 = 0$$

$$(x + 14)(x - 10)$$

$$x = -14 \text{ (discard) or } x = 10$$

$$\text{side lengths} = (10+6)(10-2)$$

$$= 16 \text{ and } 8$$

3. $y^2 - 3y - 10 = 0$

$$y = 5, y = -2$$

4. $3x^2 + 8x - 16 = 0,$

$$3x^2 + 12x - 4x - 16 = 0$$

$$3x(x + 4) - 4(x + 4) = 0$$

$$(3x - 4)(x + 4) = 0$$

$$x = 4/3, -4$$

5. $2a^2 - 3a - 9 = 0$

$$2a^2 - 6a + 3a - 9 = 0$$

$$2a(a - 3) + 3(a - 3) = 0$$

$$(2a + 3)(a - 3) = 0$$

$$a = 3/2, 3$$

6. $10x^2 - 27x - 9 = 0$

$$10x^2 - 30x + 3x - 9 = 0$$

$$10x(x - 3) + 3(x - 3) = 0$$

$$(10x + 3)(x - 3) = 0$$

$$x = -3/10, 3$$

7. $15y^2 - 10y + 6y - 4 = 0$

$$5y(3y - 2) + 2(3y - 2) = 0$$

$$(5y + 2)(3y - 2) = 0, y = -2/5, 2/3$$

8. $3x^2 + 12x - 10x - 40$

$$3x(x + 4) - 10(x + 4)$$

$$(3x - 10)(x + 4)$$

$$\text{possible values } x = 10/3, -4$$

9. $\frac{6x}{2x - 1}$

10. $\frac{x - 1}{5x}$

11. $\frac{5x - 1}{(x + 1)(x - 2)}$

12. $\frac{36x - 23}{12}$

13. $\frac{-4}{3}$

14. $x < -2$

15. $y > -3$

16. $6x+5 + 8x+3 + 4x-2 + 10x+1 + AB$

$$28x + 7 + AB = 32x - 7$$

$$AB = 4x - 14$$

17. $2(8x + 3) + 2(6x + 2) = 290$

$$16x + 6 + 12x + 4 = 290$$

$$28x + 10 = 290, x = 10$$

18. $10 \times -4 \times 3 \div (-4 + 3)^2 = -120$

19. $(2a)^3 \times 2a^k = 16a^8$

$$8a^3 \times 2a^k = 16a^8, k = 5$$

20. $36 = \frac{1}{2} \times (15 + 9) h.$

$$36 = 12 \times h,$$

$$h = 12$$

Algebra Practise

Proficiency Skill Questions - Set C

1. $4x^2 + 8x + 3 = 0, x = -1.5.$

Possible factorised equations

$$(2x + 1)(2x + 3) \times$$

$$(4x + 2)(x + 1.5) \checkmark$$

Other solution = -0.5

2. $V = 50 \times (w + 10) \times w$

$$V = 50w^2 + 500w$$

$$60 \text{ litres} = 60,000\text{cm}^3$$

$$50w^2 + 500w = 60000$$

$$50w^2 + 500w - 60000 = 0$$

$$50(w^2 + 10w - 1200) = 0$$

$$50(w + 40)(w - 30) = 0$$

$$w = 30, w+10 = 40, \text{height} = 50 \text{ (cm)}$$

3. Perimeter = 30 cm the length of each side = 10 cm (equilateral triangle)

$$2x - y = 10$$

$$2y + x = 10 \text{ or } x = 10 - 2y$$

Substitute $2(10 - 2y) - y = 10$

$$20 - 4y - y = 10$$

$$20 - 5y = 10$$

$$-5y = -10, y = 2$$

If $y = 2$ then $2x - 2 = 10$

$$2x = 12, x = 6$$

4.
$$\frac{(x - 2y)(x + 2y)}{x(x - 2y)} = \frac{x + 2y}{x}$$

5.
$$\frac{5x + 6x}{10} = \frac{11x}{10}$$

6. $x^2 + 2x - 255 = 0$

$$(x + 17)(x - 15), x = -17 \text{ or } x = 15$$

7.
$$\frac{8m + 3m}{12} = \frac{11m}{12}$$

8. $Z = \text{Zahara's age: } Z(Z + 4) = 725$

$$Z^2 + 4Z - 725 = 0$$

$$(Z + 29)(Z - 25) = 0$$

$$Z = -29 \text{ or } Z = 25$$

$$\text{Zahara} = 25, \text{Maddox} = 29$$

Zahara is now 5

= another 20 years.

9. a. $T = V - dx$

$T = \text{Total volume remaining}$

$V = \text{initial volume}$

$d = \text{number of drippers}$

$x = \text{amount used by each dripper}$

b. $T = V - dx$

$$60 = 150 - 4x$$

$$4x = 90$$

$$x = 22.5$$

Amount of water used by each dripper is 22.5 litres.

10. $4x + 3x^2 + 5x - 2x^2 = 22$

$$9x + x^2 = 22$$

$$x^2 + 9x - 22 = 0$$

$$(x + 11)(x - 2) = 0$$

$$x = -11 \text{ or } x = 2$$

Width of path = 2m

Algebra Practise

Advanced Skill Questions - Set A

1. $x^2 + 6x + c = 0$

1 solution means $(x + a)^2 = (x + 3)^2$
equation is $x^2 + 6x + 9 = 0$

2. $A = \text{distance Andy}, B = \text{distance Billy}$

$$A + B = 8 \text{ or } B = 8 - A$$

$$B = \frac{15}{10} \text{ Billy travels 1.5km faster}$$

$$1.5A = 8 - A, A = 3.2, B = 4.8$$

3. $\frac{x^2 - 1}{x^2 + 2x + 1} = \frac{3}{4}$

$$\frac{(x-1)(x+1)}{(x+1)(x+1)} = \frac{3}{4} \quad (\text{but } x \neq -1)$$

$$4x - 4 = 3x + 3, x = 7$$

4. $S + A = 38$

$$10s + 12A = 420$$

$$10s + 10A = 380 \text{ (equation } 1 \times 10)$$

$$2A = 40$$

$$\text{Adults} = 20, \text{Students} = 18$$

5. $4 \times 2^x = 2^{6x+3}$

$$2^2 \times 2^x = 2^{6x+3}$$

$$2 + x = 6x + 3 \text{ (as } A^x + A^y = A^{x+y})$$

$$5x + 1 = 0, x = -0.2 \text{ } (-1/5)$$

6. $x^2 + 5x - 36 = 144$

$$x^2 + 5x - 150 = 0$$

$$(x + 15)(x - 10) = 0$$

$$x = -15, x = 10$$

$x = 10$ as no negative lengths

7. $\frac{3}{2x} + \frac{x+4}{4}$ (cross multiply)

$$\frac{12}{8x} + \frac{2x^2 + 8x}{8x}$$

$$= \frac{2x^2 + 8x + 12}{8x}$$

8. $6x - y = 21$ or $y = 6x - 21$

$$-x + 6y = 14 \text{ or } 6y = 14 + x$$

$$36x - 126 = 14 + x$$

$$35x = 140, x = 4, y = 3$$

$$x - y = 1$$

9. $x^3 = mx,$

$$x^2 = m$$

$$x = m$$

$$x^2 = n + x \quad \text{and } m = n + x$$

$$m = n + m$$

$$n = m - \sqrt{m}$$

10. $2(12x + 1) + (4x - 2)(2x + 3) = 92$

$$24x + 2 + 8x^2 + 12x - 4x - 6 = 92$$

$$8x^2 + 32x - 96 = 0$$

$$8(x^2 + 3x - 12) = 0$$

$$8(x + 6)(x - 2) = 0$$

$$x = -6, x = 2$$

In this situation $x = 2$

11. $(x + 6)(x - 4) = x^2 + 2$

$$x^2 + 2x - 24 = x^2 + 2$$

$$2x = 2 + 24$$

$$x = 13$$

Tiles at the start = $13^2 = 169$

Algebra Practise

Advanced Skill Questions - Set B

1. $\frac{1}{2}x(4x + 6) = 35$

$$2x^2 + 3x - 35 = 0$$

$$(2x - 7)(x + 5) = 0$$

$$x = 3.5, \text{ or } x = -5$$

Disregard $x = -5$

2. $8 \times 2^{x-4} < 20$

$$2^3 \times 2^{x-4} < 20$$

$$2^{x-1} < 20$$

$$2^4 = 16 \text{ and } 2^5 = 32$$

$$x \leq 5$$

3. $A + S = 27$ and $S = 27 - A$

$$30A + 20S = 650$$

$$30A + 20(27 - A) = 650$$

$$30A + 540 - 20A = 650$$

$$10A = 110 \text{ and } A = 11, S = 16$$

$R = \text{Arabella}$ and $S = \text{Isabella}$

$$\frac{1}{2}A + \frac{2}{3}S = 16 \quad \times 6$$

$$\frac{1}{2}R + \frac{1}{3}S = 11 \quad \times 6$$

$$3R + 4S = 96$$

$$\underline{3R + 2S = 66}$$

$$2S = 30$$

$$S = 15, R = 10$$

4. $C = 2\pi r$ and $r_1 + r_2 = 10$

$$C = 2\pi r_1 + 2\pi r_2$$

$$C = 2\pi(r_1 + r_2)$$

$$C = 2\pi 10$$

$$C = 20\pi$$

5. $5^1 \times 5^{3x} = 5^{-2x^2}$

$$3x + 1 = -2x^2$$

$$2x^2 + 3x + 1 = 0$$

$$(2x + 1)(x + 1) = 0$$

$$x = -\frac{1}{2} \text{ or } x = -1$$

6. $(2y)(3y - 1) = \frac{1}{2}(2y - 2)(5y + 9)$

$$6y^2 - 2y = \frac{1}{2}(10y^2 + 8y - 18)$$

$$y^2 - 6y + 9 = 0$$

$$(y - 3)^2 = 0, y = 3$$

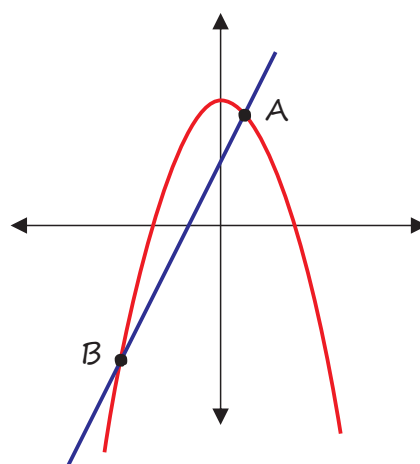
7. $7 + x - 6x^2 = 8x + 4.$

$$6x^2 + 7x - 3 = 0$$

$$(3x - 1)(2x + 3) = 0$$

$$x = -\frac{1}{3}, x = -1.5$$

$$y = 6\frac{2}{3}, y = -8$$



$$A = (-0.33, 6.67)$$

$$B = (-1, 5, -8)$$

Algebra Practise

Advanced Skill Questions - Set C

1. $5^2 \times 5^{2y+13} = 5^{y^2}$

$$2 + 2y + 13 = y^2$$

$$y^2 - 2y - 15 = 0$$

$$(y - 5)(y + 3) = 0$$

$$y = 5, y = -3$$

2. $U = \text{under } 65, V = 65 \text{ or over}$

$$U + V = 21$$

$$14U + 10V = 258$$

$$\underline{10U + 10V = 210}$$

$$4U = 48$$

$$U = \$12, V = \$9$$

3. $x^2 + cx + 16 = 0$

solution must be $(x + a)^2$

$$(x + 4)^2 = 0, c = 8$$

4. $A + B = 15$ and $B = 15 - 8$

$$B = 18/12A$$

$$B = 1.5A$$

$$1.5A = 15 - A$$

$$2.5A = 15, A = 6\text{km}, B = 9\text{km}$$

5. $D - 20 = G + 20$

$$2(G - 22) = D + 22$$

$$2G - 44 = D + 22$$

$$D = 2G - 66$$

$$2G - 66 - 20 = G + 20$$

$$G = \$106, D = 146$$

6. $A = (-5, -10), B = (9, 11)$

Using point A $-10Q = -5P + 5$ ($\times 9$)

Using point B $11Q = 9P + 5$ ($\times 5$)

$$-90Q = -45Q + 45$$

$$\underline{55Q = 45Q + 25}$$

$$-35Q = 70, Q = -2 \text{ and } P = -3$$

7. $(x - 3)(x + 3) < (x - 4)(x + 2)$

$$x^2 - 9 < x^2 - 2x - 8$$

$$-1 < -2x, x < \frac{1}{2}$$

8. $10 \times 2^{p-1} < 165$

$$2^{p-1} < 16.5$$

$$2^4 = 16 \text{ and } 2^5 = 32$$

$p < 5$ assuming p is an integer

9. $M = 5(a^2 - 3a + 4) + a^2$

$$N = (3a - 5)(2a - 4) + 7(a + 5)$$

expand and simplify

$$M = 6a^2 - 15a + 20$$

$$N = 6a^2 - 15a + 55$$

$$\text{then } M = N - 35$$

10. $3^3 \times 3^{x-4} > 200$

$$3^{x-1} > 200 \quad (3^5 = 243), x \geq 6$$

11. $A - 2 = B + 2$ or $A = B + 4$

$$B - 2 = (A + 2) \div 2$$

$$B - 2 = (B + 4 + 2) \div 2$$

$$2B - 4 = B + 6,$$

$$B = 10, A = 14$$

Algebra Practise

Advanced Skill Questions - Set D

1. $x^4 - 18x^2 + 81 = 0$

$$(x^2 - 9)(x^2 - 9) = 0$$

$$(x^2 - 9) = (x + 3)(x - 3)$$

$$x = 3 \text{ or } -3$$

2. $Ry - 2Rx = 5y - 4x$

$$4x - 2Rx = 5y - Ry$$

$$x(4 - 2R) = y(5 - R)$$

$$x = \frac{y(5 - R)}{4 - 2R}$$

3. Perimeter A $6W + 8L = 20$

Perimeter B $8W + 10L = 26$

multiply A $\times 8$ and B $\times 6$

$$48W + 64L = 160$$

$$\underline{48W + 60L = 156}$$

$$4L = 4, L = 1, W = 2$$

Rec A Length = 4, width = 6

Rec B, Length = 5, width = 8

4. $(3 + 2x)(2 + 2x) = 2(2 \times 3)$

$$6 + 6x + 4x + 4x^2 = 12$$

$$4x^2 + 10x - 6 = 0$$

$$2x^2 + 5x + 3 = 0 \quad (2 \times 3 = 6)$$

$$2x^2 + 6x - 1x - 3 = 0$$

$$2x(x + 3) - 1(x + 3) = 0$$

$$(2x - 1)(x + 3) = 0$$

$$x = -\frac{1}{2}, x = -3 \text{ (ignore)}$$

5. $(3x + A)^2 = 9x^2 + Bx + 16, A = 4$

$$(3x + 4)(3x + 4)$$

$$9x^2 + 24x + 16, A = 4, B = 24$$

6. $(2^3)^x \times (2^2)^{x^2 - 6} = 2^2$

$$2x^2 + 3x + 14 = 0 \quad (2 \times 14 = 28)$$

$$2x^2 + 7x - 4x - 14 = 0$$

$$(x - 2)(2x + 7) = 0, x = 2 \text{ or } -7/2$$

7. $3x + 2y = 20$

$$2y = 20 - 3x$$

$$xy = 14, x \times \frac{20 - 3x}{2} = 14$$

$$20x - 3x^2 = 28$$

$$3x^2 - 20x + 28 = 0 \quad (3 \times 28 = 84)$$

$$3x^2 - 6x - 14x + 28 = 0$$

$$(3x - 14)(x - 2) = 0, x = 14/3 \text{ or } 2$$

8. $(3x - 5)^2 \leq 3x^2 + 1$

$$6x^2 - 30x + 24 \leq 0$$

$$x^2 - 5x + 4 \leq 0, 1 \leq x \leq 4$$

9. $(-3, 6) \quad -6A = -3B + 6$

$$(3, 10) \quad \underline{10A = 3B + 6}$$

$$4A = 12, A = 3, B = 8$$

10. $x^2 + 4x - 140 = 0$

$$(x + 14)(x - 10) = 0, x = 10$$

11. $6^2 \times 6^{2x+6} = 6^{x^2}$

$$2x + 8 = x^2, x = 4 \text{ or } x = -2$$

12. Any odd number = $2n + 1$

Next consecutive odd = $2n + 3$

Using $n^2 - 2$.

$$[(2n+3)^2 - 2] - [(2n+1)^2 - 2]$$

$$= 4(2n + 2)$$

4 is a factor and it is therefore divisible by 4.

Algebra Practise

Advanced Skill Questions - Set E

1. 2 points where the ball is 3m above the ground

$$3 = 4x - x^2$$

$$x^2 - 4x + 3 = 0, x = 3 \text{ or } x = 1$$

This means the ball is 3m or above for 2m of horizontal distance.

Finding the total horizontal distance zero metres or above.

$$x(x - 4) = 0, x = 0 \text{ or } x = 4$$

Total horizontal distance = 4m

$$\% \text{ distance} = 2\text{m} \div 4\text{m} (50\%)$$

2. $2n^2 - n + 5$ (consecutive = $n+1$)

$$2(n+1)^2 - (n+1) + 5$$

$$2n^2 + 3n + 6$$

$$- \underline{2n^2 - n + 5}$$

$$4n + 1$$

or if $n = 1, 2, 3, 4, 5$

$$2n^2 - n + 5 = 6, 11, 20, 33, 50$$

Difference = 5, 9, 13, 17

$$\text{Rule} = 4n + 1$$

3. $x^2 + 6x + c = 0$

solution must have form $(x + a)^2$

$$(x + 3)^2 \text{ and } c = 9$$

Final equation $x^2 + 6x + 9 = 0$.

4. $x^2 - ax - 24 = 0$

factors of 24: -6, 4; -12, 2; -8, 3

$$(x - 12)(x + 2) = 0, x = 12, x = -2$$

5. $R = 5n^2 + n - 4 - 4n^2 - 2n + 1$

$$R = n^2 - n - 3$$

$$= (n^2 - n + 5) - 8$$

$$S = R - 8$$

6. Substitute point G into equation

$$13 = A(-1)^2 + B(-1) + 2$$

$$13 = A - B + 2$$

$$A - B = 11$$

Substitute point H into equation

$$18 = A(4)^2 + B(4) + 2$$

$$18 = 16A + 4B + 2$$

$$16A + 4B = 16$$

Simultaneous Equation

$$4A - 4B = 44$$

$$\underline{16A + 4B = 16}$$

$$20A = 60, A = 3, B = -8$$

7. Area $(4x + 2)(6x + 3)$

$$= 24x^2 + 24x + 6$$

$$= 6(4x^2 + 4x + 1)$$

8. $(y + 3)^2 = \frac{A(x^2 - 7)}{B}$

$$B(y + 3)^2 = A(x^2 - 7)$$

$$\frac{B(y + 3)^2}{A} + 7 = x^2$$

$$x = \sqrt{\frac{B(y + 3)^2 + 7}{A}}$$

Algebra Practise

Advanced Skill Questions - Set F

1. $3x - y = 8$

$$\frac{(2^3)^x}{2^y} = \frac{2^{3x}}{2^y}$$

$$= 2^{3x-y} \quad (2^8) = 256$$

2a. $(\sqrt{3} + \sqrt{15})^2 = 3 + 2\sqrt{3}\sqrt{15} + 15$

$$= 18 + 2\sqrt{45}$$

$$= 18 + 2 \times \sqrt{9} \times \sqrt{5}$$

$$= 18 + 6\sqrt{5}$$

2b. $(3 + \sqrt{5})^2 = 9 + 3\sqrt{5} + 3\sqrt{5} + 5$

$$= 14 + 6\sqrt{5}$$

$$k^2 + 14 + 6\sqrt{5} = 18 + 6\sqrt{5}$$

$$k^2 = 4$$

$$k = 2 \text{ (ignore } -2)$$

3. $x+y+z = \frac{m+9}{2} + \frac{2m+15}{2} + \frac{3m+18}{2}$

$$= \frac{6m+42}{2}$$

$$= 3m + 21$$

$$= \frac{3m+21}{3}$$

$$= m + 7$$

4. $3x + 1 = x + 4 - x^2$

$$x^2 + 2x - 3 = 0$$

$$(x+3)(x-1), x = -3 \text{ or } 1,$$

coordinates $(-3, 8), (1, 4)$

5. $2x(2x-1) = \frac{1}{2}(2x-4)(7x+9)$

$$4x^2 - 2x = (x-2)(7x+9)$$

$$4x^2 - 2x = 7x^2 - 5x - 18$$

$$3x^2 - 3x - 18 = 0$$

$$(3x+6)(x-3) = 0$$

$$x = -2 \text{ (ignore) or } x = 3$$

6. Let $u = 3^x$ and $u^2 = 3^{2x}$

$$u^2 - 4u - 45 = 0$$

$$(u-9)(u+5) = 0, u = 9 \text{ or } u = -5$$

$$9 = 3^2 \text{ and } 81 = 3^4 \text{ then } x = 2$$

$$-5 = 3^x \text{ and } 25 = 3^{2x} \text{ x not +integer}$$

7. $3^2 \times 3^{x+y} = (3^3)^{2x}$

$$2 + x + y = 6x$$

$$y = 5x - 2$$

8. $= (2n)^2 + (2n+2)^2 + (2n+4)^2 - 2$

$$= 12n^2 + 24n + 18$$

$$= 2(6n^2 + 12n + 9), 2 \text{ is a factor}$$

$$= 3(4n^2 + 8n + 6), 3 \text{ is a factor}$$

$$= 6(2n^2 + 4n + 3), 6 \text{ is a factor}$$

9. $y = (2a-1)^2 - (2b-1)^2$ then

$$y = 4a^2 - 4a - 4b^2 + 4b$$

$$y = 4(a-b)(a+b-1)$$

10. $n(2n+6) > n(n^2+3)$

$$2n+3 > n^2+3$$

$$n^2 - 2n - 3 < 0$$

$$(n-3)(n+1) < 0$$

$$n-3 < 0, n < 3 \text{ \& } n+1 < 0, n < -1$$

$$\text{therefore } -1 < n < 3$$

11. $33\pi = 2\pi r(r+9.5)$

$$33 = 2r^2 + 19r$$

$$2r^2 + 19r - 33 = 0$$

$$(2r-3)(r+11) = 0$$

$$r = 1.5 \text{ or } r = 11 \text{ (ignore)}$$

1 - Substitution

1. $a = 3, b = -2, c = 13$ and $z = \frac{1}{2}$
 - a. $abc - 3b \rightarrow 3 \times (-2) \times 13 - 3 \times -2 = -72$
 - b. $b^2 + 4c - 3z \rightarrow (-2)^2 + 4 \times 13 - 3 \times \frac{1}{2} = 54.5$
 - c. $(a^4 + b + \frac{1}{2}c) \div z \rightarrow (3^4 + (-2) + \frac{1}{2} \times 13) \div \frac{1}{2} = 171$

2. $h = 2g^2 + 0.15d - 5l; g = 6, d = 8, l = 10$

$$h = 2(6)^2 + 0.15 \times 8 - 5 \times 10$$

$$h = 23.2$$

3. $C = 80 + 75h; h = 5\frac{1}{2}$

$$C = 80 + 75 \times 5.5$$

$$C = \$492.50$$

4. $A = LW; L = 5 \text{ cm}, W = 14.25 \text{ cm}$

$$A = 5 \times 14.25$$

$$A = 71.25 \text{ cm}^2$$

5. $P = 2(W + L); L = 5 \text{ cm}, W = 14.25 \text{ cm}$

$$P = 2(14.25 + 5)$$

$$P = 38.5 \text{ cm}$$

6. $V = \pi r^2 h; r = 6 \text{ cm}, h = 14 \text{ cm}$

$$V = \pi \times 6^2 \times 14$$

$$V = 1583.4 \text{ cm}^3, (\text{or } 504\pi \text{ cm}^3)$$

7. $SA = 2\pi r^2 + 2\pi rh; r = 6 \text{ cm}, h = 14 \text{ cm}$

$$SA = 2 \times \pi \times 6^2 + 2 \times \pi \times 6 \times 14$$

$$SA = 754 \text{ cm}^2 (\text{or } 240\pi \text{ cm}^2)$$

8. $E = \frac{1}{2}mv^2; v = 45 \text{ metres per second}, m = 210\,000 \text{ kg}$

$$E = \frac{1}{2} \times 210000 \times 45^2$$

$$E = 212\,625\,000$$

9. $M = 0.0625s + s; s = \$2000$

$$M = 0.0625 \times 2000 + 2000$$

$$M = \$2125$$

10. $z^2 = 11 + x^2; x = -5.$

$$z^2 = 11 + (-5)^2$$

$$z = \sqrt{36}$$

$$z = 6$$

2 - Expanding Single Brackets and Simplifying Expressions

Expand each expression

1. $3(5x - 8) = 15x - 24$
2. $xy(10 + z) = 10xy + xyz$
3. $-5(y - 2) = -5y + 10$
4. $-3a(12 - 2b) = -36a + 6ab$
5. $-xy(3 - y) = -3xy + xy^2$
6. $5b(abc + 8) = 5ab^2c + 40b$
7. $-4(-3 + c) = 12 - 4c$
8. $-6r(2 - 3r) = -12r + 18r^2$
9. $3xy(x + 7) = 3x^2y + 21xy$
10. $xy(y + 18) = xy^2 + 18xy$
11. $8xy(2xy - xy) = 16x^2y^2 - 8x^2y^2$
12. $4x^2(y^3 + 2x^3) = 4x^2y^3 + 8x^5$
13. $3(x + 1) + 4(4x + 2)$
 $= 3x + 3 + 16x + 8$
 $= 19x + 11$
14. $4(3 - x) - 3(2x + 1)$
 $= 12 - 4x - 6x - 3$
 $= 9 - 10x$
15. $3(5x - 0.5) - 4x(-y + 0.25)$
 $= 15x - 1.5 + 4xy - x$
 $= 14x + 4xy - 1.5$

Simplify each expression

1. $5x + x = 6x$
2. $3a + 4b + 4a = 7a + 4b$
3. $7a + 5b - 16a + 12b$
 $= -9a + 17b$
4. $4x^2 + x^2 + 3x - 11x$
 $= 5x^2 - 8x$ or $x(5x - 8)$
5. $6ab - 3a - 3b + 4b$
 $= 6ab - 3a + b$
 or $3a(2b - 1) + b$
6. $7pq + 10pq^2 - pq + pq^2$
 $= 6pq + 11pq^2$ or $pq(6 + 11q)$
7. $3x - 4x^2 + 9x - 10x^2 =$
 $= 12x - 14x^2$ or $2x(6 - 7x)$
8. $b^3 - 10ab^3 + 8b^3$
 $= 9b^3 - 10ab^3$ or $b^3(9 - 10a)$
9. $-5z^3 - 2z - 10z + 3z - 4z^3$
 $= -9z^3 - 9z$ or $9z(z^2 - 1)$
10. $x^2 + 9x - 52$
 $= (x + 13)(x - 4)$
11. $-9t^2 + t - 21t^2$
 $= -30t^2 + t$ or $t(-30t + 1)$
12. $2x^2 - 11x^2 - 21$
 $= -9x^2 - 21$
13. $x - x - x + 0.5x - 3x$
 $= -3.5x$
14. $3x(2 + y) + 7x(2 + y)$
 $= 6x + 3xy + 14x + 7xy$
 $= 20x + 10xy$
15. $-5x^2 - (-2x) - (-5x^2) + 8x$
 $= 10x$

3 - Simplifying Expressions and Expanding Single Brackets

Simplify each expression:

$$1. \quad 40y \div 10y = 4$$

$$2. \quad 30x^2 \div 5x^2 = 6$$

$$3. \quad 3d \times 7d^2 = 21d^3$$

$$4. \quad 6e^3f^2 \times 4ef = 24e^4f^3$$

$$5. \quad 3x^3 \div 12x^2 = \frac{x}{4}$$

$$6. \quad 3pq^2 \times 5p^4q^{-1} = 15p^5q$$

$$7. \quad 30d^4 \times 4cd^2 = 120cd^6$$

$$8. \quad 200x^2yz^3 \div 100x^3y^3z^2 = 2x^{-1}y^{-2}z$$

$$= \frac{2}{x} \cdot \frac{1}{y^2} \cdot z$$

$$9. \quad -12a^4c \times -3a^4c^2 = 36a^8c^3$$

$$10. \quad 2(0.5x + 2.7x) - 3x = 3.4x$$

$$11. \quad 70x \div 140x^2 = \frac{1}{2x}$$

$$12. \quad 50x^3y^5z^2 \times 3x^2yz^4 = 150x^5y^6z^6$$

$$13. \quad 21a^3b^{-4}c^{-2} \div 28a^{-4}b^5c^{-1} = \frac{3a^7}{4b^9c}$$

$$14. \quad 30y^2 \times 5y^{-3} = 150y^{-1}$$

$$= \frac{150}{y}$$

$$15. \quad 15z^3 \times 15z^{-4.5} = 225y^{-1.5}$$

$$= \frac{225}{y^{1.5}}$$

Expand each expression:

$$1. \quad x(x + y) = x^2 + xy$$

$$2. \quad 4a(a - a^2) = 4a^2 - 4a^3$$

$$3. \quad 2bc(3bc + 2b) = 6b^2c^2 + 4b^2c$$

$$4. \quad 5x(x^2 - xy) = 5x^3 - 5x^2y$$

$$5. \quad bc(a + b^2) = abc + b^3c$$

$$6. \quad x^2y(y + xy) = x^2y^2 + x^3y^2$$

$$7. \quad 3yz(-3y - z) = -9y^2z - 3yz^2$$

$$8. \quad 8x(x^3 + x^2y) = 8x^4 + 8x^3y$$

$$9. \quad a^3(7a - a^2b) = 7a^4 - a^5b$$

$$10. \quad 4x(2x^4 - 3x^2) = 8x^5 - 12x^3$$

$$11. \quad xz(x^2z - xyz) = x^3z^2 - x^2yz^2$$

$$12. \quad x(a + bc) = ax + bcx$$

$$13. \quad x^2(2x + y) - x(x^2 - xy)$$

$$= x^3 + 2x^2y$$

$$14. \quad x^3(2x - 4) + x^2(x^2 - 2)$$

$$= 3x^4 - 4x^3 - 2x^2$$

4 - Simple Factorising and Expanding Brackets

Factorise each expression:

1. $5x + 5 = 5(x + 1)$
2. $20x + 12 = 4(5x + 3)$
3. $6xy - 12x = 6x(y - 2)$
4. $5x + 10 + 20xy = 5(x + 2 + 4xy)$
5. $9x^2 + 5x = x(9x + 5)$
6. $3x^2y + y^2x = xy(3x + y)$
7. $32xy + 40x = 8x(4y + 5)$
8. $15y + 45xyz = 15y(1 + 3xz)$
9. $12x - 16 = 4(3x - 4)$
10. $60y + 12y^2 = 12y(5 + y)$
11. $8x^2 + 16x^5 = 8x^2(1 + 2x^3)$
12. $abc^4d - a^2bc^3 = ab(c^4d - ac^3)$
13. $3xy^2 - 4xy + xy = xy(3y - 3)$
14. $9a^3b^2 + 3ab = 3ab(3a^2b + 1)$
15. $x^2 + (xy)^2 = x^2 + x^2y^2$
 $= x^2(1 + y^2)$

Expand each expression:

1. $(x + 8)(x + 4)$
 $= x^2 + 12x + 32$
2. $(x + 20)(x + 6)$
 $= x^2 + 26x + 120$
3. $(p - 1)(4 - p)$
 $= -p^2 + 5p - 4$
4. $(a + 9)(a - 7)$
 $= a^2 + 2a - 63$
5. $(3y + 3)(y + 2)$
 $= 3y^2 + 9y + 6$
6. $(2x - 5)^2$
 $= 4x^2 - 20x + 25$
7. $(9x + 2)(9x - 2)$
 $= 81x^2 - 4$
8. $(2x - 5)(x - 4)$
 $= 2x^2 - 13x + 20$
9. $(3x - 7)(5x + 6)$
 $= 15x^2 - 17x - 42$
10. $(2x + 3)^2$
 $= 4x^2 + 12x + 9$
11. $(4x + 3)(4x - 3)$
 $= 16x^2 - 9$
12. $(2 - x)(2 + x)$
 $= 4 - x^2$
13. $(ax + b)(ax - 2b)$
 $= a^2x^2 - axb - 2b^2$
14. $(\frac{1}{2}x + \frac{3}{4})^2$
 $= \frac{4x^2 + 12x + 9}{16}$
15. $(x + 2)(4x - 9)$
 $= 4x^2 - x - 18$

5 - Factorise Quadratics and Solving Equations

Factorise these expressions

$$1. \quad x^2 + 5x + 6 = (x + 3)(x + 2)$$

$$2. \quad x^2 - 10x + 25 = (x - 5)(x - 5)$$

$$3. \quad 3x^2 - 30x + 27 = 3(x^2 - 10x + 9) \\ = 3(x - 9)(x - 4)$$

$$4. \quad x^2 + 2x - 8 = (x + 4)(x - 2)$$

$$5. \quad 2x^2 - 4x - 30 = 2(x^2 - 2x - 15) \\ = 2(x - 5)(x + 3)$$

$$6. \quad x^2 + 12x + 27 = (x + 9)(x + 3)$$

$$7. \quad 2x^2 + 32x + 128 = 2(x^2 + 16x + 64) \\ = 2(x + 8)^2$$

$$8. \quad x^2 + 8x + 16 = (x + 4)^2$$

$$9. \quad 3x^2 - 24x + 48 = 3(x^2 - 8x + 16) \\ = 3(x - 4)^2$$

$$10. \quad x^2 + 22x - 48 = (x + 24)(x - 2)$$

$$11. \quad 5x^2 - 15x + 10 = 5(x^2 - 3x + 2) \\ = 5(x - 2)(x - 1)$$

$$12. \quad x^2 - 0.25 = (x + 0.5)(x - 0.5)$$

$$13. \quad 50 - 2x^2 = 2(5 + x)(5 - x)$$

$$14. \quad 3x^2 + x - 4 = (3x + 4)(x - 1)$$

$$15. \quad 10x^2 - 6 + 11x = (5x - 2)(2x + 3)$$

Solve each equation:

$$1. \quad 2x + 6 = x + 3 \\ x = -3$$

$$2. \quad 3x + 6 = 48 \\ x = 14$$

$$3. \quad 4x - 8 = 5x - 2 \\ x = -6$$

$$4. \quad 6x + 7 = 2x + 20 \\ x = 3.25$$

$$5. \quad x + 6 = 2x - 8 \\ x = 14$$

$$6. \quad 3(x + 2) = 5(x - 2) \\ x = 8$$

$$7. \quad 4 = 8 - \frac{x}{3} \\ x = 12$$

$$8. \quad 3 - \frac{x}{3} = -5 \\ x = 24$$

$$9. \quad 5x - 6 = -39 \\ x = -6.6 \text{ or } \frac{-33}{5}$$

$$10. \quad 2 - \frac{x}{2} = -1 - \frac{x}{4} \\ x = 12$$

6 - Practise the Basics of "Expand" and "Factorise"

1. $u(u + 1) = u^2 + u$
2. $v(v - 6) = v^2 - 6v$
3. $-w(3w - 2) = -3w^2 + 2w$
4. $x(4x + 5) = 4x^2 + 5x$
5. $3y(2y - 3) = 6y^2 - 9y$
6. $-z(-5z + 3) = 5z^2 - 3z$
7. $3 + 2(x - 8) = 3 + 2x - 16$
 $= 2x - 13$
8. $5(x + 7) - 12 = 5x + 35 - 12$
 $= 5x + 23$
9. $3(x - 6) + 2(4x - 5)$
 $= 3x - 18 + 8x - 10$
 $= 11x - 28$
10. $4(a + 6) - 2(a - 2)$
 $= 4a + 24 - 2a + 4$
 $= 2a + 28$
11. $2x(x + 1) - x(7 - x)$
 $= 2x^2 + 2x - 7x + x^2$
 $= 3x^2 - 5x$
12. $x^2(x + 1) = x^3 + x^2$
13. $\frac{1}{2}(4x + 12) = 2x + 6$
14. $\frac{3}{4}(12x - 6) = 9x - 4.5$
15. $3x(2x^2 - 4) = 6x^3 - 12x$
16. $x(x^2 + 4) + x(3x + 2)$
 $= x^3 + 4x + 3x^2 + 2x$
 $= x^3 + 3x^2 + 6x$
17. $6x + 24 = 6(x + 4)$
18. $5x - 25 = 5(x - 5)$
19. $11x^2 - 66x = 11x(x - 6)$
20. $10x + 25xy = 5x(2 + 5y)$
21. $100x + 20y = 20(5x + y)$
22. $27 - 33x = 3(9 - 11x)$
23. $5x^2 + x = x(5x + 1)$
24. $6a^2 + 3a = 3a(2a + 1)$
25. $15b^2 - 30b = 15b(b - 2)$
26. $14y^2 + 21y = 7y(2y + 3)$
27. $5 + 5n^2 = 5(1 + n^2)$
28. $6x^2 + 18xy = 6x(x + 3y)$
29. $2xy - 4ab = 2(xy - 2ab)$
30. $3p^2 - 9pq = 3p(p - 3q)$
31. $(x + 1)(x + 6) = x^2 + 7x + 6$
32. $(x + 2)(x + 8) = x^2 + 10x + 16$
33. $(x - 5)(x + 7) = x^2 + 2x - 35$
34. $(x - 2)(x + 9) = x^2 + 7x - 18$
35. $(x + 4)(x - 5) = x^2 - x - 20$
36. $(x + 7)(x - 3) = x^2 + 4x - 21$
37. $(x - 10)(x - 15) = x^2 - 25x + 150$
38. $(x - 8)(x - 11) = x^2 - 19x + 88$
39. $(x + 6)^2 = x^2 + 12x + 36$
40. $(x - 9)^2 = x^2 - 18x + 81$
41. $(x + 1)^2 + 10 = x^2 + 2x + 11$
42. $(x - 5)^2 - 20 = x^2 - 10x + 5$
43. $x^2 + 10x + 21 = (x + 7)(x + 3)$
44. $x^2 + x - 12 = (x + 4)(x - 3)$
45. $x^2 - 2x - 15 = (x - 5)(x + 3)$
46. $x^2 - 14x + 40 = (x - 10)(x - 4)$
47. $x^2 + 11x + 30 = (x + 6)(x + 5)$
48. $x^2 + x - 2 = (x + 2)(x - 1)$
49. $x^2 - 3x - 10 = (x - 5)(x + 2)$
50. $x^2 - 4x - 96 = (x - 12)(x + 8)$
51. $x^2 - 5x - 14 = (x - 7)(x + 2)$
52. $x^2 - 16 = (x - 4)(x + 4)$
53. $x^2 - 81 = (x - 9)(x + 9)$
54. $(x - 3)^2 - 16 = x - 6x + 9 - 16$
 $= x - 6x - 7$
 $= (x - 7)(x + 1)$
55. $x^2 + 2x - 15 = (x + 5)(x - 3)$
56. $x^2 - 6x + 8 = (x - 4)(x - 2)$
57. $2x^2 - 2x - 220 = 2(x^2 - x - 110)$
 $= 2(x - 11)(x + 10)$
58. $4x^2 - 100 = 4(x^2 - 25)$
 $= 4(x - 5)(x + 5)$

7 - Solving Equations

1. Equations $2S = A$, $P - 2 = S$, $P = 11$

Peter = 11, Sue = 9, Ali = 18

2. $2x + 10 = 42$

$2x = 32$, $x = 16$

3. equations $J = 2 + M$, $P = 2M$, $J + M + P = 6$

Philip = 2 hours, Mary = 1 hour, John = 3 hours

4. $(x) + (x+1) + (x+2) + (x+3) = 114$

$4x + 6 = 114$

$x = 27$

5. Area = $(x + 18)(x - 5)$ and Perimeter $2(x - 5) + 2(x + 18) = 54$

$4x + 26 = 54$, $x = 7$

Dimensions are 2cm and 25cm

6. $(2x + 1)(2y + 1) = 4xy + 2x + 2y + 1$

$= 2(2xy + x + y) + 1$

It is a multiple of 2 and adding 1 at the end shows it must be odd

7. Consecutive numbers $= a-2, a-1, a, a+1, a+2$

Summing the numbers together $(a-2) + (a-1) + a + (a+1) + (a+2)$

$= 5a$

8. Consecutive numbers $= a, a+1, a+2$

Square and sum each. $= a^2 + (a+1)^2 + (a+2)^2$

$= a^2 + a^2 + 2a + 1 + a^2 + 4a + 4$

$= 3a^2 + 6a + 5$

Subtract 2, divide the result by 3 $= 3a^2 + 6a + 3$

$= a^2 + 2a + 1$

$= (a + 1)^2$

9. Sung's birth month equation is: $4x + (12 - x) - 2(5 + x) = 10$

Simplifying gives $x + 2 = 10$ or $x = 8$

Kim's birth month is 8 (August)

10. Because the graph is quadratic the highest point will be at the mid way point between where the water starts and finishes i.e. when $x = 25$ metres

Therefore $0.5(25) - 0.01(25)^2$

$= 12.5 - 0.01(625)$

$= 12.5 - 6.25$

$= 6.25$ metres high

8 - Solving Quadratic Equations

Solve these equations:

1. $(x + 3)(x - 2) = 0$

$x = 3, x = 2$

2. $(x + 0.5)(4x + 6) = 0$

$x = -0.5, x = -1.5$

3. $(2x + 8)(x - 4) = 0$

$x = -4, x = 4$

4. $(4x - 10)(3x + 12) = 0$

$x = 2.5, x = -4$

5. $(4x - 20)(2x + 2.5) = 0$

$x = 5, x = -1.25$

6. $2x(x - 4) = 10$

$x = -1, x = 5$

7. $(4x + 2)(2x - 6) = 0$

$x = -\frac{1}{2}, x = 3$

8. $3x(2x + 4) = 0$

$x = 0, x = 2$

9. $x^2 + 5x = -6$

$x = -2, x = -3$

10. $(x - 4)(x + 5) = 0$

$x = 4, x = -5$

11. $5x^2 - 3x = 0$

$x = 0.6, x = 0$

12. $6x^2 - 2 = 0$

$x = 0.58, x = -0.58$

13. $100 - 4x^2 = 0$

$x = 5, x = -5$

14. $2x^2 - 6x + 4 = 0$

$x = 2, x = 1$

15. $(x + 3)(x - 2) = 14$

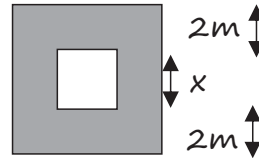
$x = 4, x = -5$

16. $2x^2 + 5x = -3$

$2x^2 + 5x + 3 = 0 \quad (2x + 3)(x + 1) = 0$

$x = -1.5, x = -1$

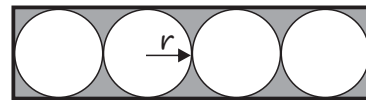
17.



$(x + 4)(x + 4) - x^2 = 40$

$x = 3$

18.



total circle area = $4\pi r^2$

$8r \times 2r - 4\pi r^2 = 14$

$16r^2 - 4\pi r^2 = 14$

$r^2(16 - 4\pi) = 14$

$r = 2.02$

19. $\frac{1}{2}(2h + 6) \times h = 40$

$h^2 + 3h - 40 = 0$

$(h + 8)(h - 5) = 0$

$h = -8, h = 5$

20. Area Front Square + Area Triangle

$= (\text{length})^2 + (\frac{1}{2} \times \text{base} \times \text{height})$

$= x^2 + \frac{1}{2} \cdot x \cdot \frac{1}{2}x$

$= x^2 + \frac{1}{4}x^2$

$= \frac{5}{4}x^2$

Cross Sectional Area (CSA) = $\frac{5}{4}x^2$

Volume = CSA \times Length

$40 = \frac{5}{4}x^2 \times 4x$

$40 = 5x^3$

$8 = x^3,$

therefore $x = 2 \text{ m}$

The Algebra Project

9 - Algebraic Fractions

Simplify

$$1. \quad x - \frac{x-2}{2} \qquad \frac{x+2}{2}$$

$$2. \quad \frac{x}{2} - \frac{2x-1}{2} \qquad \frac{-x+1}{2}$$

$$3. \quad \frac{x^2 - 3x - 28}{x+4} \qquad x - 7$$

$$4. \quad \frac{2p^2 - 12pq}{6p^2} \qquad \frac{p-6q}{3p}$$

$$5. \quad \frac{x^2 - 4x - 5}{x^2 + 6x + 5} \qquad \frac{x-5}{x+5}$$

$$6. \quad \frac{2}{3a} \times \frac{a^2}{12a} \qquad \frac{1}{18}$$

$$7. \quad \frac{5a}{2b} \div \frac{a}{6b} \qquad 15$$

$$8. \quad \frac{3ab^2 - 4a^3b + ab^2}{4ab^2} \qquad \frac{b-a^2}{b}$$

$$9. \quad \frac{3x}{7} + \frac{2x}{5} \qquad \frac{29x}{35}$$

$$10. \quad \frac{9x^3}{12x^2} \qquad \frac{3x}{4}$$

$$11. \quad \frac{x^2 - 5x + 6}{x-3} \qquad x - 2$$

$$12. \quad \frac{8x^2}{20x - 12x^2} \qquad \frac{2x}{5-3x}$$

Solve

$$1. \quad \frac{x+2}{3} - \frac{2x-1}{5} = 1$$

$$x = -2$$

$$2. \quad \frac{3}{y-4} - \frac{3}{y+4} = \frac{2}{y+1}$$

$$y = 14, y = -2$$

$$3. \quad \frac{x}{5} - \frac{2x-1}{2} \geq \frac{-3x}{5}$$

$$x \leq \frac{5}{2}$$

$$4. \quad \frac{2-5x}{4} > 3$$

$$x < -2$$

$$5. \quad \frac{4x-6}{3} > 2x+1$$

$$x < \frac{-9}{2}$$

$$6. \quad 6x-3 \geq 8x+9$$

$$x \leq -6$$

10 - Quadratic Sequences

Give an expression for the n th term.

1. 6, 9, 14, 21, 30 = $n^2 + 5$
2. 11, 14, 19, 26, 35 = $n^2 + 10$
3. -1, 2, 7, 14, 23 = $n^2 - 2$
4. -5, -2, 3, 10, 19 = $n^2 - 6$
5. 4, 10, 18, 28, 40 = $n^2 + 3n$
6. 9, 20, 33, 48, 65 = $n^2 + 8n$
7. 5, 10, 17, 26, 37 = $n^2 + 2n + 2$
8. 3, 3, 5, 9, 15, 23 = $n^2 - 3n + 5$
9. 2, 10, 20, 32, 46, 62 = $n^2 + 5n - 4$
10. -10, -11, -10, -7, -2, 5 = $n^2 - 4n - 7$

11. 3, 8, 15, 24, 35, 48 = $n^2 + 2n$
 2nd difference = 2 therefore n^2
 3, 8, 15, 24, 35, 48
 n^2 1, 4, 9, 16, 25, 36
 difference 2 4 6 8 12 10
 difference each time = 2 therefore $+ 2n$

12. $\frac{1}{2}, \frac{4}{3}, \frac{9}{4}, \frac{16}{5}$

Numerator 1, 4, 9, 16 ... n^2

Denominator 2, 3, 4, 5, ... $n + 1$

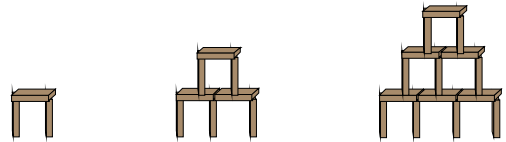
n th term = $\frac{n^2}{n + 1}$

13. Block base = 3, 4, 5 $n + 2$
 Block height = 4, 5, 6 $n + 3$
 Area $(n + 2)(n + 3) = n^2 + 5n + 6$

14. Write the n th term for:

- a. 5, 12, 21, 32 $n^2 + 4n$
- b. 3, 9, 17, 27 $n^2 + 3n - 1$

15.



Slabs = 24, 35, 48, 63, 80

Look at the pattern between the storeys and the slabs.

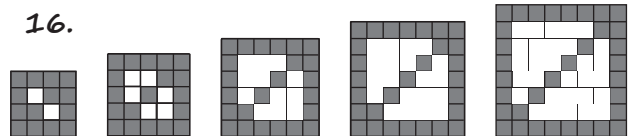
$1 \times 3 = 3, 2 \times 4 = 8, 3 \times 5 = 15, 4 \times 6 = 24, 5 \times 7 = 35$ therefore

$y = x(x + 2)$

$y = x^2 + 2x$

Using this formula, 25 storeys would need 675 slabs of wood.

16.



Total = Grey + white

$n^2 = 5n - 6 + \text{white}$

white = $n^2 - 5n + 6$

17. Sequence Q = 3, 8, 15, 24, 35

The n th term of the sequence is

$n^2 + kn$ where k represents a number. Find the value of k .

Using term 1, $n^2 + kn = 3$

$1^2 + k = 3$

$k = 2$